Sadagopan Rajesh

TEST TRIGONOMETRY

AUG 08, 2024

Maximum time: 30 *minutes*

KEM6 - Advanced Maths for Std 11 only @ *ABIMS*

Try on your own ! Don't use calculators ! Think and Answer !

Name:____ Standard:__________

1. $8.\sin\dfrac{x}{8}.\cos\dfrac{x}{2}.\cos\dfrac{x}{4}.\cos\dfrac{x}{8} =$

   A. $\cos x$. B. $8\cos x$. C. $8\sin x$. D. $\sin x$.

2. $ABCD$ is a *quadrilateral* such that $\sin A = \sin B = \sin C$.

   Which of the following is <u>definitely</u> *true*?

   A. $\cos A = \cos B = \cos C$. B. $\tan A = \tan B = \tan C$.

   C. $cosecA = cosecB = cosecC$. D. All of these

3. $co\sec 10^\circ - \sqrt{3}\sec 10^\circ \ =$

   A. 1 B. 4 C. 0 D. none of these

4. If $\sin 203^\circ \cos 22^\circ + \cos 203^\circ.\sin 22^\circ \ = \ k$, then the value of $100k^2 =$________.

5. $\dfrac{\sin\left(A+B+C\right) + \sin A \sin B \sin C - \cos A \cos B \sin C}{\sin\left(A+B\right)} \ =$

   A. $\cos C$ B. $\sin C$ C. $tanC$ D. none of these

6. If $A + B = \dfrac{3\pi}{4}$,then $\left(1+\cot A\right).\left(1+\cot B\right) \ = $ ______.

7. The *value* of $\cot 44^\circ \times \left(\cot 76^\circ + \cot 16^\circ\right) - \cot 76^\circ.\cot 16^\circ =$ ________.

8. The *value* of $8\left(\cos^2 73^\circ + \cos^2 47^\circ + \cos 73^\circ.\cos 47^\circ\right) =$ ________.

Sadagopan Rajesh | COMBO TEST 1 | SEPTEMBER 05, 2024

Maximum time: 135 *minutes*

Basics from previous standards, Sequences and Series
Sets; Relations and Functions, Trigonometry, Mathematical Induction

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Name:______________________________ Standard:____________

I Answer the following questions accordingly !

I.I Section - A : Questions on *Concepts*

1. A is set of letters in the word $'INTELLIGENT'$. Then, $|A| =$

   A. 7 B. 10 C. 11 D. none of these

2. Let $A = \{1, 2, 3, 4\}$ and $B = \{1, 3, 5, 7, 9, 11\}$.

   Which of the following could be a function f from $A \to B$?

   A. $f(x) = \{(1, 1), (2, 3), (3, 7), (4, 11)\}$
   B. $f(x) = \{(1, 3), (2, 5), (3, 9)\}$
   C. $f(x) = \{(5, 1), (1, 2), (2, 7), (3, 9), (4, 11)\}$
   D. $f(x) = \{(1, 3), (2, 5), (3, 7), (3, 9), (4, 11)\}$

3. Which of the following is the next term of the sequence :

$$1, 3, 5, 7, 9, 11, 13, 15, 17, 19, \_\_\_\_?$$

   A. 31 B. 21 C. 20 D. cannot exactly determine

4. ABC is a triangle. Which of the following is definitely true?

   A. $\cos A > 0$ B. $\sin A > 0$ C. $\tan A > 0$ D. All of these

5. There are infinitely many values of θ for which $\cos\theta = -1$.

   The general value of θ is given by

   A. $\theta = n\pi$, where $n \in \mathbb{Z}$ B. $\theta = (2n+1)\pi$, where $n \in \mathbb{Z}$

   C. $\theta = (2n-1)\pi$, where $n \in \mathbb{R}$ D. none of these

6. The sum of the first n terms of a finite A.P :

$$25, 22, 19, ..., l$$

   is 116. Then the value of $(l - n)$ is

   A. 4 B. 8 C. -4 D. none of these

7. If the fourth term of a G.P is square of its second term and the first term is -3, then the $7th$ term is

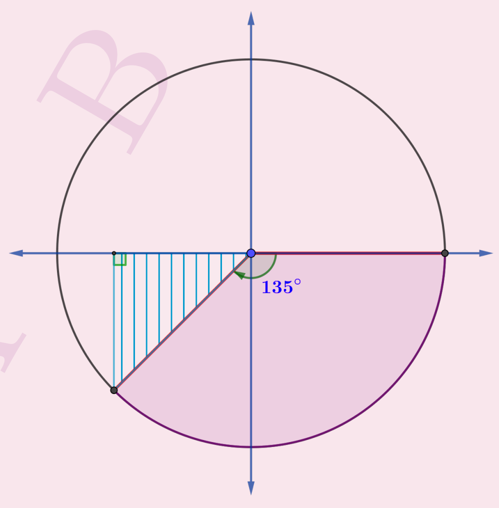
   A. -1096 B. -2340 C. -2187 D. -3120

8. Let $A = \{1, 2\}$ and $B = \{3, 4\}$.

   The maximum number of relations that can be defined from $A \to B$ is

   A. 4 B. 8 C. 16 D. none of these

9. $\cos(-135°)$=



   A. $-\sqrt{2}$ B. $\frac{1}{\sqrt{2}}$ C. $\frac{-1}{\sqrt{2}}$ D. -1

10. If $\sin\theta = \frac{7}{25}$, then the value of $\sec\theta$ = ______. Note : $\theta \in \left(\frac{\pi}{2}, \pi\right)$

   A. $\frac{24}{25}$ B. $\frac{25}{24}$ C. $\frac{-24}{25}$ D. $\frac{-25}{24}$

11. Which of the following is true?

A. $\sin 3\theta = 2\sin\frac{3\theta}{2}.\cos\frac{3\theta}{2}$

B. $\sin 3\theta = 3\sin\theta - 4\sin^3\theta$

C. $\sin 3\theta = \pm\sqrt{\frac{1-\cos 6\theta}{2}}$

D. All of these

12. Let $S_1 = \sum_{k=1}^{10} = 1^3 + 2^3 + 3^3 + \cdots + 10^3$

Let $S_2 = \sum_{k=1}^{10} = 1 + 2 + 3 + \cdots + 10$

Let $S_3 = \sum_{k=1}^{10}(k-1)k(k+1) = 0\times 1\times 2 + 1\times 2\times 3 + \cdots + 9\times 10\times 11$

Then, which of the following is true?

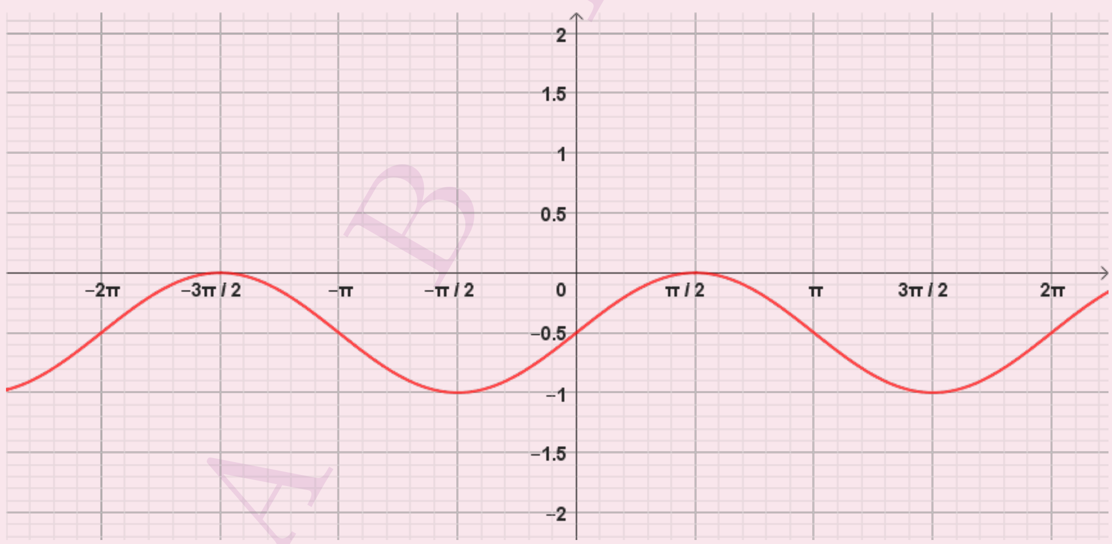
A. $S_3 = S_1 - S_2$

B. $S_3 = \frac{S_1 + S_2}{2}$

C. $S_3 = S_1 + S_2$

D. None of these

13. Which of the following function is shown in the graph?



A. $\frac{1}{2}\sin x - \frac{1}{2}$

B. $\sin x - \frac{1}{2}$

C. $\cos x - 1\frac{1}{2}$

D. none of these

14. If a, b, c, d are in H.P , then the value of $\left(\frac{1}{a^2} - \frac{1}{d^2}\right) \div \left(\frac{1}{b^2} - \frac{1}{c^2}\right)$ is

A. 4

B. 3

C. 2

D. 1

15. If A and B are two sets, then $A - (A - B)$ is

A. ϕ

B. B

C. $A \cup B$

D. $A \cap B$

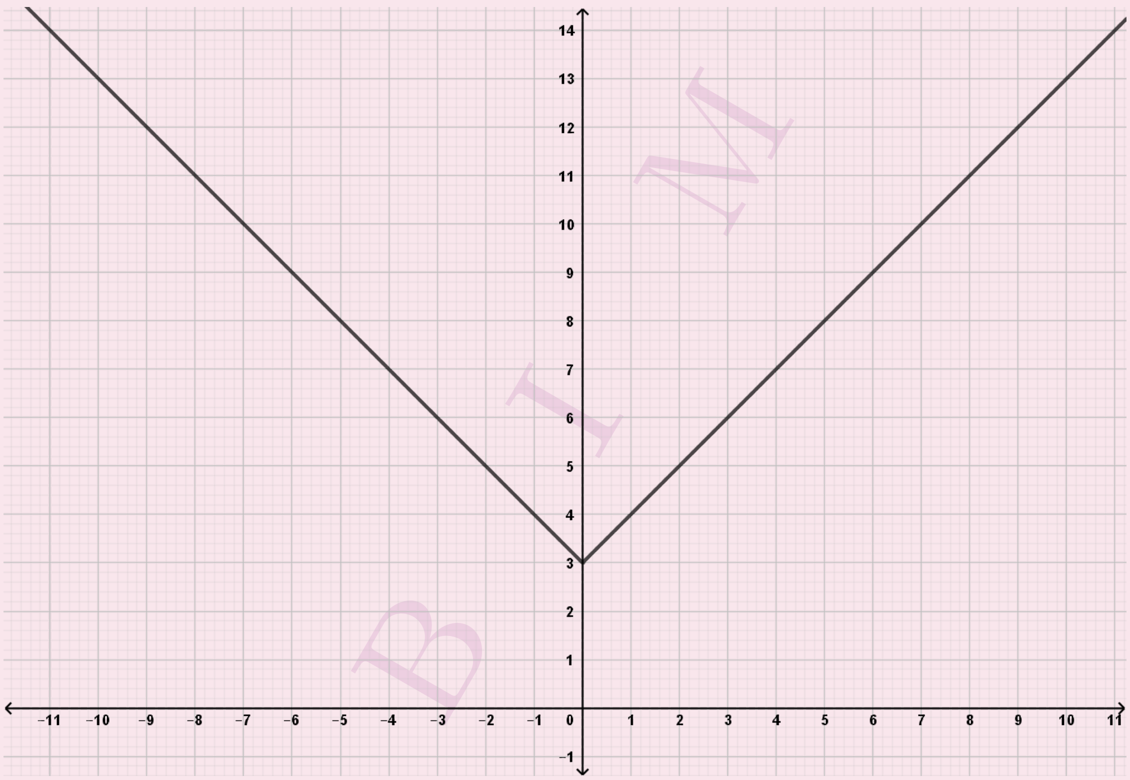
16. Let set $A = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$ and set $B = \{10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20\}$

Let $R : A \to B = \Big\{(a, b) : a \in A, b \in B, a \text{ is a factor of } b\Big\}$ be the relation from set A to set B.

Then, $|R : A \to B| =$

A. 11 B. 64 C. 31 D. none of these

17. Which of the following function is shown in the graph?



A. $|x + 3|$ B. $|x - 3|$ C. $|x| + 3$ D. none of these

18. If a, b, c, d are in G.P , then $a^2 - b^2, b^2 - c^2, c^2 - d^2$ are in

A. A.P B. G.P C. neither A.P nor G.P D. none of these

19. $\sin^2 2023° + \cos^2 2024°$

A. < 1 B. $= 1$ C. > 1 D. none of these

20.

$$\sum_{k=1}^{20} \frac{1}{k(k+1)} =$$

A. $\dfrac{20}{21}$ B. $\dfrac{21}{20}$ C. $\dfrac{20}{19}$ D. $\dfrac{19}{20}$

I.II Section - B : Questions on *Applications*

21. A and B are acute angles satisfying the equations

$$\begin{cases} 3\sin^2 A + 2\sin^2 B = 1 \\ 3\sin 2A - 2\sin 2B = 0 \end{cases}$$

then $A + 2B =$

A. $\frac{\pi}{4}$ B. $\frac{\pi}{2}$ C. $\frac{3\pi}{4}$ D. $\frac{2\pi}{3}$

22. If $f(x) = \sqrt{x}, g(x) = \frac{x}{4}$ and $h(x) = 4x - 8$, then

A. $g \circ h \circ f(x) = \sqrt{x-2}$ B. $f \circ g \circ h(x) = \sqrt{x} - 2$

C. $h \circ g \circ f(x) = \sqrt{x} - 8$ D. $h \circ f \circ g(x) = \sqrt{x} - 4$

23. The sum of the first 3 terms of a G.P is 19 and their product is 216.

If the G.P. is an infinite and a convergent G.P, then the sum of all the infinite terms of the G.P is

A. 32 B. 18 C. 27 D. none of these

24. The sum of the infinite series

$$\sum_{k=1}^{\infty} \frac{(2k-1)(2k+1)}{2^k} = \frac{1 \times 3}{2} + \frac{3 \times 5}{2^2} + \frac{5 \times 7}{2^3} + \cdots \infty$$

is

A. 49 B. 23 C. 64 D. none of these

25. If $\sin x \cos y = \frac{1}{4}$ and $3\tan x = 4\tan y$,then $\sin(x + y)$=

A. $\frac{1}{4}$ B. $\frac{3}{4}$ C. 1 D. $\frac{7}{16}$

26. If in $\triangle ABC$, $\frac{2\cos A}{a} + \frac{\cos B}{b} + \frac{2\cos c}{c} = \frac{a}{bc} + \frac{b}{ca}$, then

A. $\angle A = 90^\circ$ B. $\angle B = 90^\circ$ C. $\angle C = 90^\circ$ D. none of these.

27. Let $f(x)$ be defined on $[-2, 2]$ and is given by

$$f(x) = \begin{cases} -1 & -2 \le x \le 0 \\ x - 1 & 0 < x \le 2 \end{cases}$$

and $g(x) = f(|x|) + |f(x)|$, then $g(x) =$

A. $\begin{cases} -x & -2 \le x < 0 \\ 0 & 0 \le x < 1 \\ x - 1 & 1 \le x \le 2 \end{cases}$ *B.* $\begin{cases} -x & -2 \le x < 0 \\ 0 & 0 \le x < 1 \\ 2(x-1) & 1 \le x \le 2 \end{cases}$

C. $\begin{cases} -x & -2 \le x < 0 \\ x-1 & 0 < x \le 2 \end{cases}$ D. none of these

28. Of the members of three athletic teams in a school, 21 are in the cricket team,26 are in the hockey team, 29 are in the football team. Among them,14 play hockey and cricket,15 play hockey and football, and 12 play football and cricket. Eight play all the three games.

The total number of members in the 3 athletic teams is

A. 43 B. 76 C. 49 D. none of these

29. If the roots of the cubic equation $x^3 - 7x^2 + cx - 8 = 0$ are in G.P,then the value of c is

A. 11 B. 12 C. 13 D. 14

30. The sum

$$\sum_{k=1}^{100} \frac{k}{k^4 + k^2 + 1} = \frac{N}{10101}.$$

Then, the value of N is

A. 5050 B. 5100 C. 4900 D. none of these.

I.III Section - C : Questions on *Applications*

31. Then n^{th} term of a G.P is 128 and the sum of its n terms is 255.

If the common ratio is 2, then its first term is ____.

32.

$$\sum_{k=1}^{10} k(k+3)^2 = 1 \times 4^2 + 2 \times 5^2 + 3 \times 6^2 + \cdots + 10 \times 13^2$$

is ____

33. If

$$\sum_{k=1}^{\infty} \frac{1}{17^k} \times \underbrace{33333.....333}_{k \;\; digits} = S$$

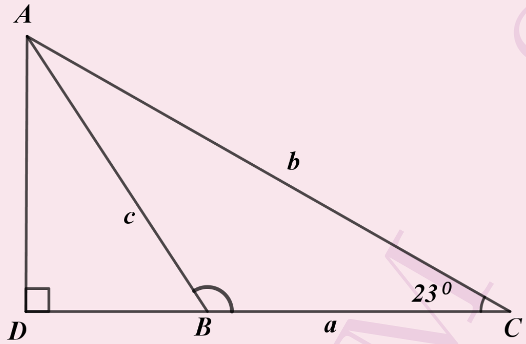
then the value of $112 \times S$ is ____

34. If $\cos x + \sin x = \sqrt{2} \cos x$, then the value of $\tan^2 x + 2 \tan x =$ ____

35. The value of $\tan\left(\frac{\pi}{8}\right) - \cot\left(\frac{\pi}{8}\right) =$ ____

36. If $\cos^3\theta + \cos^3(120° + \theta) + \cos^3(240° + \theta) = y\cos 3\theta$,then the value of $4y$ is ____

37. In $\triangle ABC, AD$ is the altitude from A, as shown.



Given $b > c, \angle C = 23°$ and $AD = \dfrac{abc}{b^2 - c^2}$ in $\triangle ABC$, then $\angle ABC =$ ____ (in degrees)

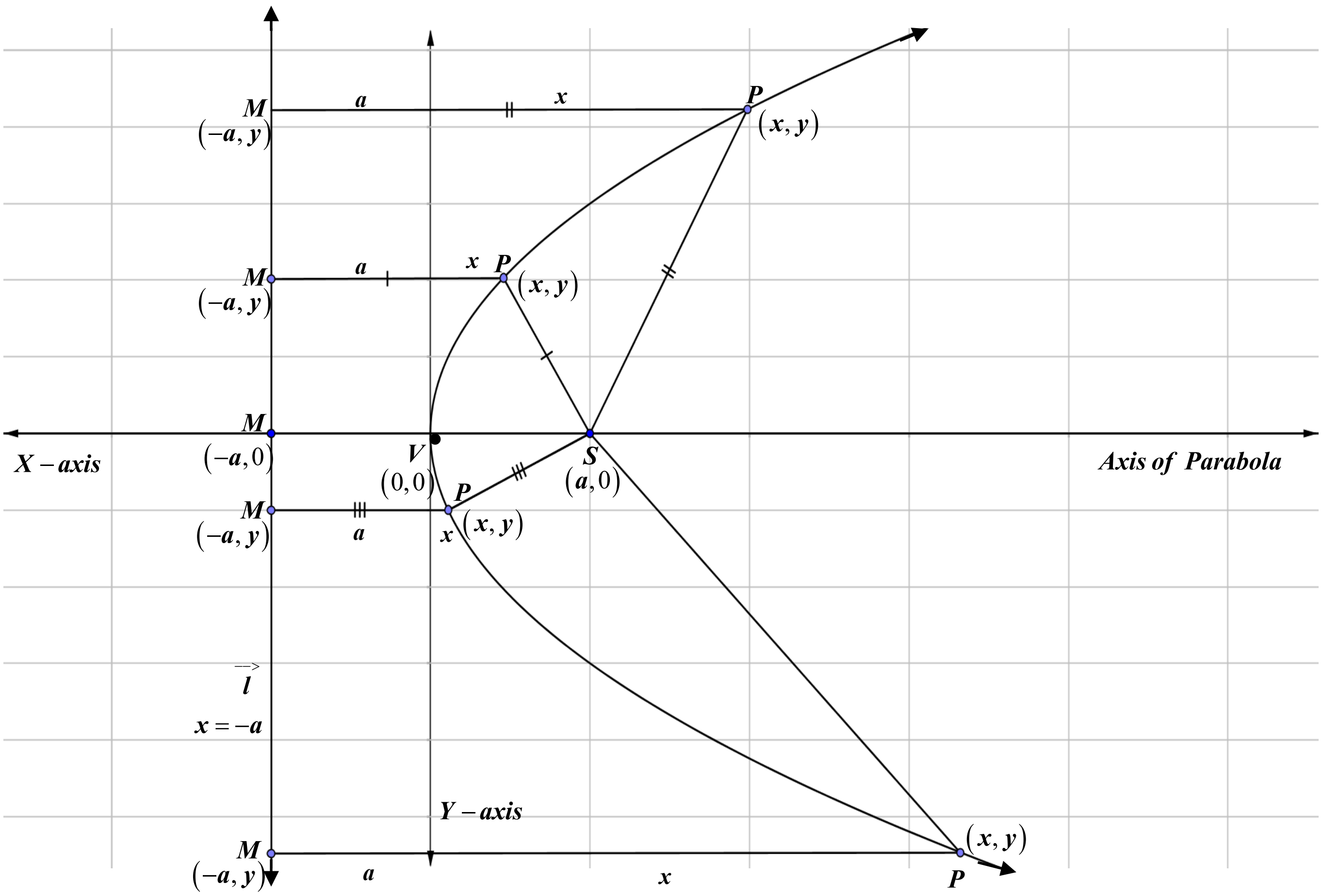
38. If $f(x)$ is a function such that $2f(x^2) + 3f\left(\dfrac{1}{x^2}\right) = x^2 - 1$ for all $x > 0$, then, $15f(3) =$ ____.

39. Let $V = \{a, e, i, o, u\}, V - B = \{e, o\}, B - V = \{k\}$. Then,$|B| =$ ____.

40. If $f(x) = \sqrt{|x|^2 - 5|x| + 6} + \sqrt{2 - |x| - |x|^2}$ is real, then the domain of $f(x)$ is $[a, b]$ where $b - a =$ ____.

Basic Forms of Parabolas

by Shri Sadagopan Rajesh



$S(a,0)$ is *a fixed point* called *Focus,* where $a > 0$.

$\vec{l} : x = -a$ is *a fixed line* called *Directrix.*

$P(x, y)$ is *a moving point* in the *same plane* as $S, \vec{l}$ tracing a *Parabola.*

$M(-a, y)$ is the *foot of the perpendicular* from P to $\vec{l}$.

M varies its position relatively with P.

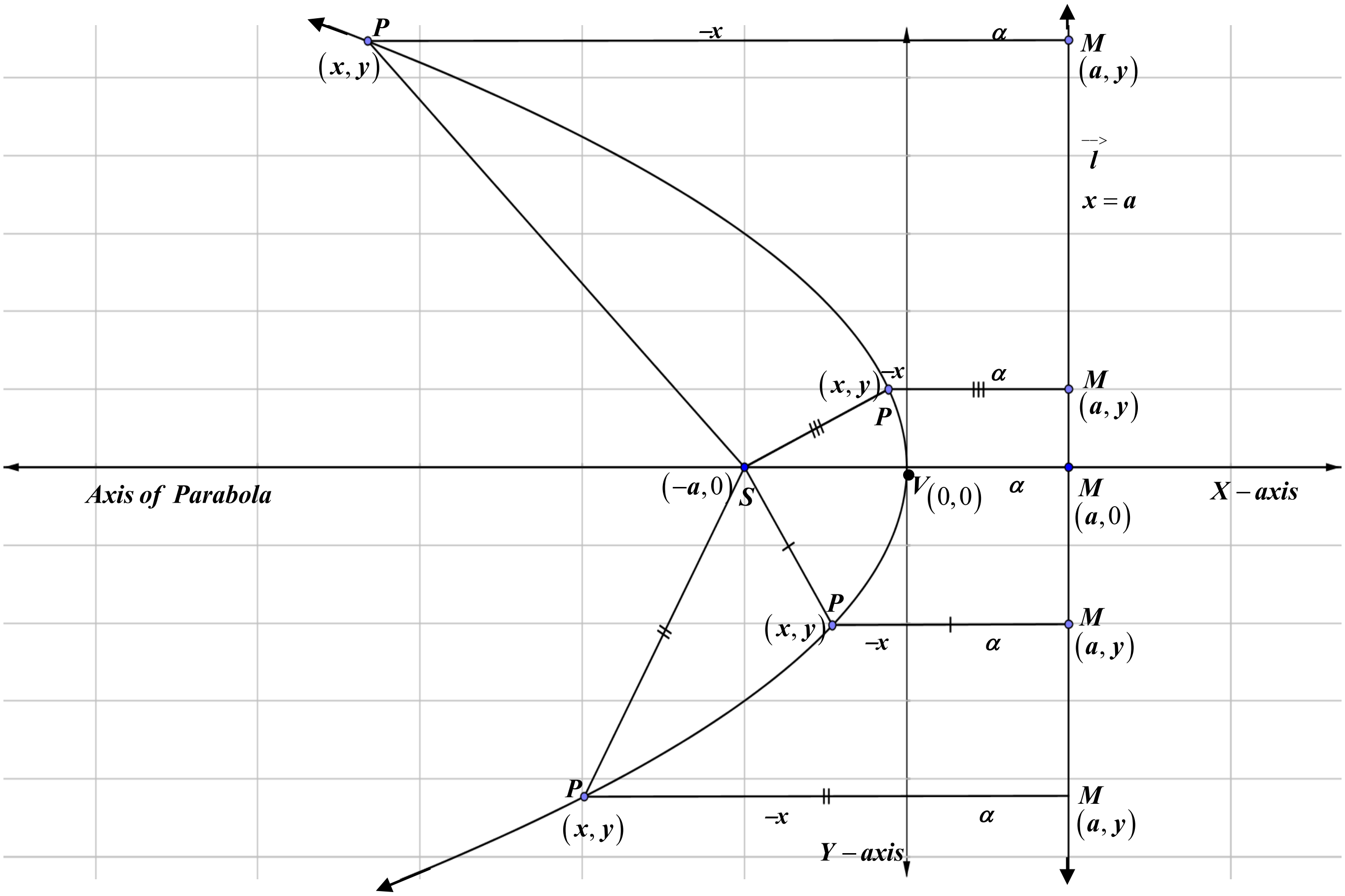
P traces a *Parabola,* such that

$$\frac{\text{distance from } P \text{ to } S}{\text{distance from } P \text{ to } \vec{l}} = e = 1.$$

That is, $\dfrac{SP}{PM} = e = 1.$

$$SP^2 = PM^2 \Rightarrow (x-a)^2 + (y-0)^2 = (x+a)^2 \Rightarrow y^2 = 4ax.$$

Note: Here, from Vertical $Y-$***axis*** *to* $P(x, y)$, $\begin{cases} \text{Displacement} = x \\ \text{Distance} = x \end{cases}$ ***as*** $x > 0$.



$S(-a,0)$ is *a fixed point* called *Focus, ,* where $a > 0$.

$\vec{l}$: $x = a$ is *a fixed line* called *Directrix.*

$P(x,y)$ is *a moving point* in the *same plane* as S, $\vec{l}$ tracing a *Parabola.*

$M(a,y)$ is the *foot of the perpendicular* from P to $\vec{l}$.

M varies its position relatively with P .

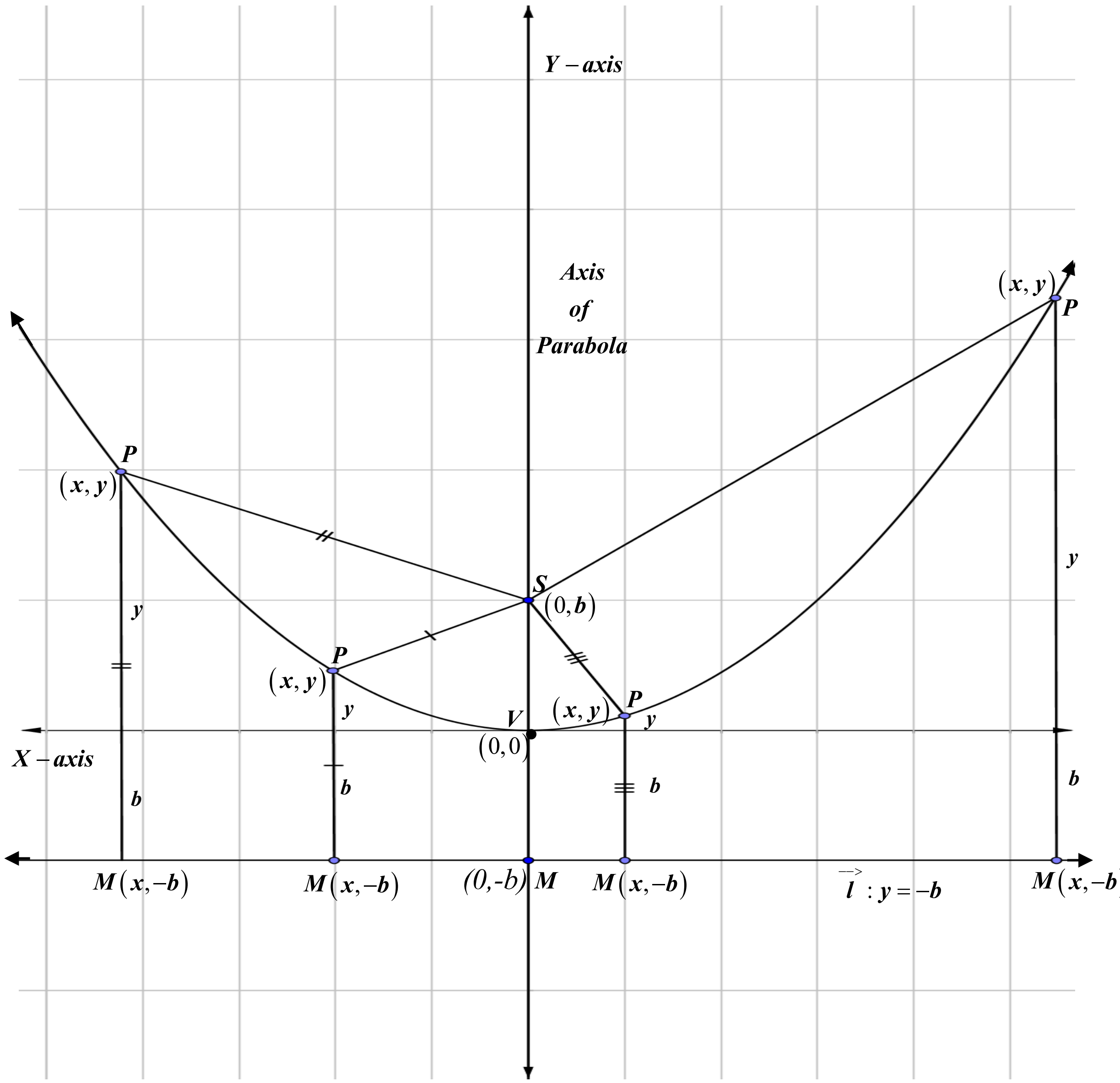
P traces a *Parabola,* such that

$$\frac{\text{distance from } P \text{ to } S}{\text{distance from } P \text{ to } \vec{l}} = e = 1.$$

That is, $\dfrac{SP}{PM} = e = 1.$

$$SP^2 = PM^2 \Rightarrow (x+a)^2 + (y-0)^2 = (-x+a)^2 \Rightarrow y^2 = -4ax.$$

Note: Here, from Vertical Y− ***axis*** *to* $P(x,y)$, $\begin{cases} \text{Displacement} = x \\ \text{Distance} = -x \end{cases}$ ***as*** $x < 0$.



$S(0,b)$ is *a fixed point* called *Focus, ,* where $b>0$.

$\vec{l}: y=-b$ is *a fixed line* called *Directrix.*

$P(x,y)$ is *a moving point* in the *same plane* as $S, \vec{l}$ tracing a *Parabola.*

$M(x,-b)$ is the *foot of the perpendicular* from P to $\vec{l}$.

M varies its position relatively with P.

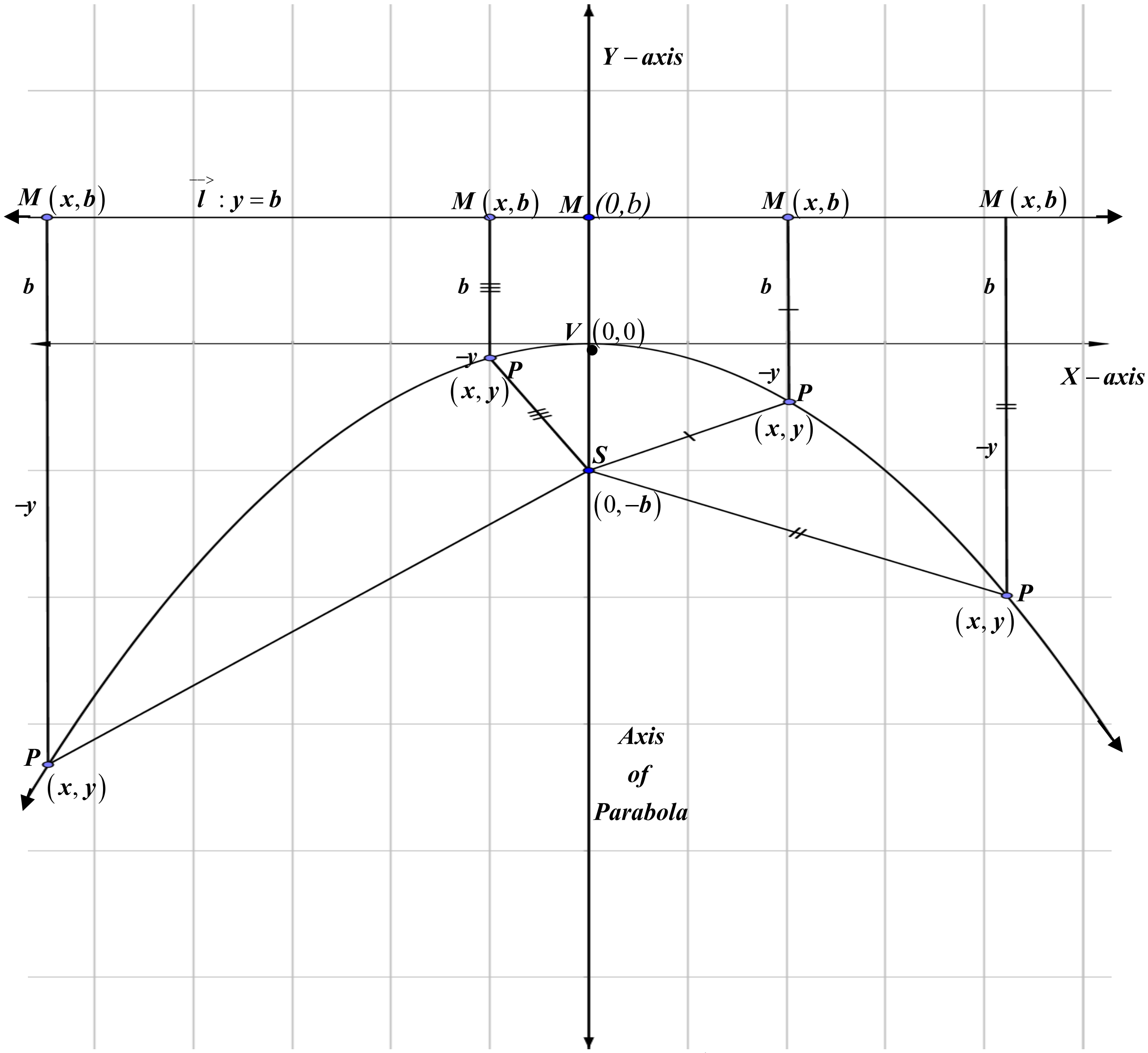
P traces a *Parabola,* such that

$$\frac{\text{distance from } P \text{ to } S}{\text{distance from } P \text{ to } \vec{l}} = e = 1.$$

That is, $\frac{SP}{PM} = e = 1.$

$$SP^2 = PM^2 \Rightarrow (x-0)^2 + (y-b)^2 = (y+b)^2 \Rightarrow x^2 = 4by.$$

Note: Here, from Vertical X− ***axis*** *to* $P(x,y)$, $\begin{cases} \text{Displacement} = y \\ \text{Distance} = y \end{cases}$ ***as*** $y>0$.



$S(0,-b)$ is *a fixed point* called *Focus, ,* where $b>0$.

$\vec{l}: y=b$ is *a fixed line* called *Directrix.*

$P(x,y)$ is *a moving point* in the *same plane* as $S, \vec{l}$ tracing a *Parabola.*

$M(x,b)$ is the *foot of the perpendicular* from P to $\vec{l}$.

M varies its position relatively with P.

P traces a *Parabola,* such that

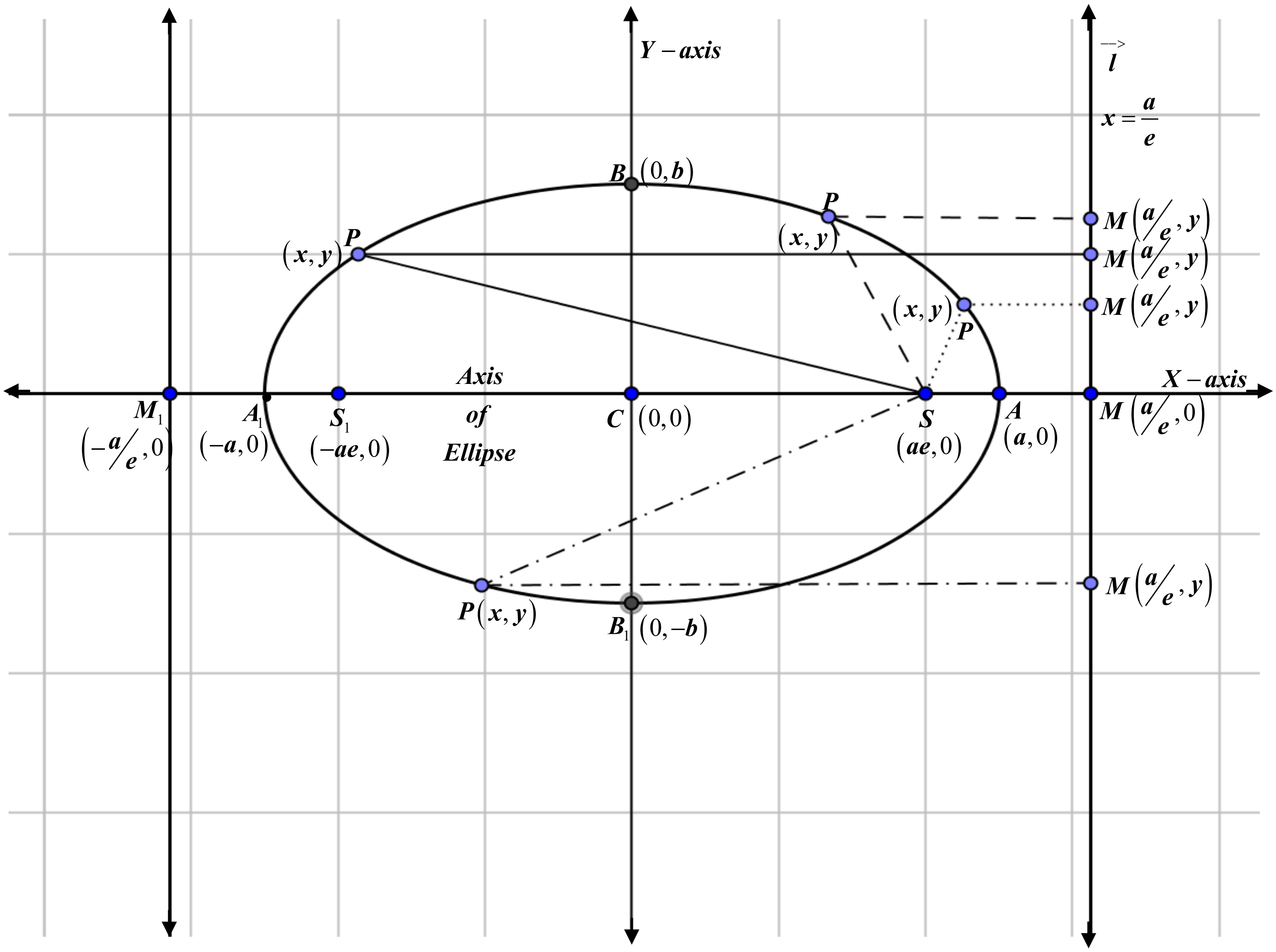
$$\frac{\text{distance from } P \text{ to } S}{\text{distance from } P \text{ to } \vec{l}} = e = 1.$$

That is, $\frac{SP}{PM} = e = 1.$

$$SP^2 = PM^2 \Rightarrow (x-0)^2 + (y+b)^2 = (-y+b)^2 \Rightarrow x^2 = -4by.$$

Note: Here, from Vertical X$-$***axis*** *to* $P(x,y)$, $\begin{cases} \text{Displacement} = y \\ \text{Distance} = -y \end{cases}$ ***as*** $y<0$.

Basic Forms of Ellipses



$S(ae, 0)$ is *a fixed point* called *Focus, ,* where $a > 0$.

$\vec{l}: x = \dfrac{a}{e}$ is *a fixed line* called *Directrix.*

$P(x, y)$ is *a moving point* in the *same plane* as $S, \vec{l}$ tracing a *Ellipse.*

$M\left(\dfrac{a}{e}, y\right)$ is the *foot of the perpendicular* from P to $\vec{l}$.

M varies its position relatively with P .

P traces a *Ellipse,* such that

$$\frac{\text{distance from } P \text{ to } S}{\text{distance from } P \text{ to } \vec{l}} = e < 1.$$

That is, $\dfrac{SP}{PM} = e < 1.$

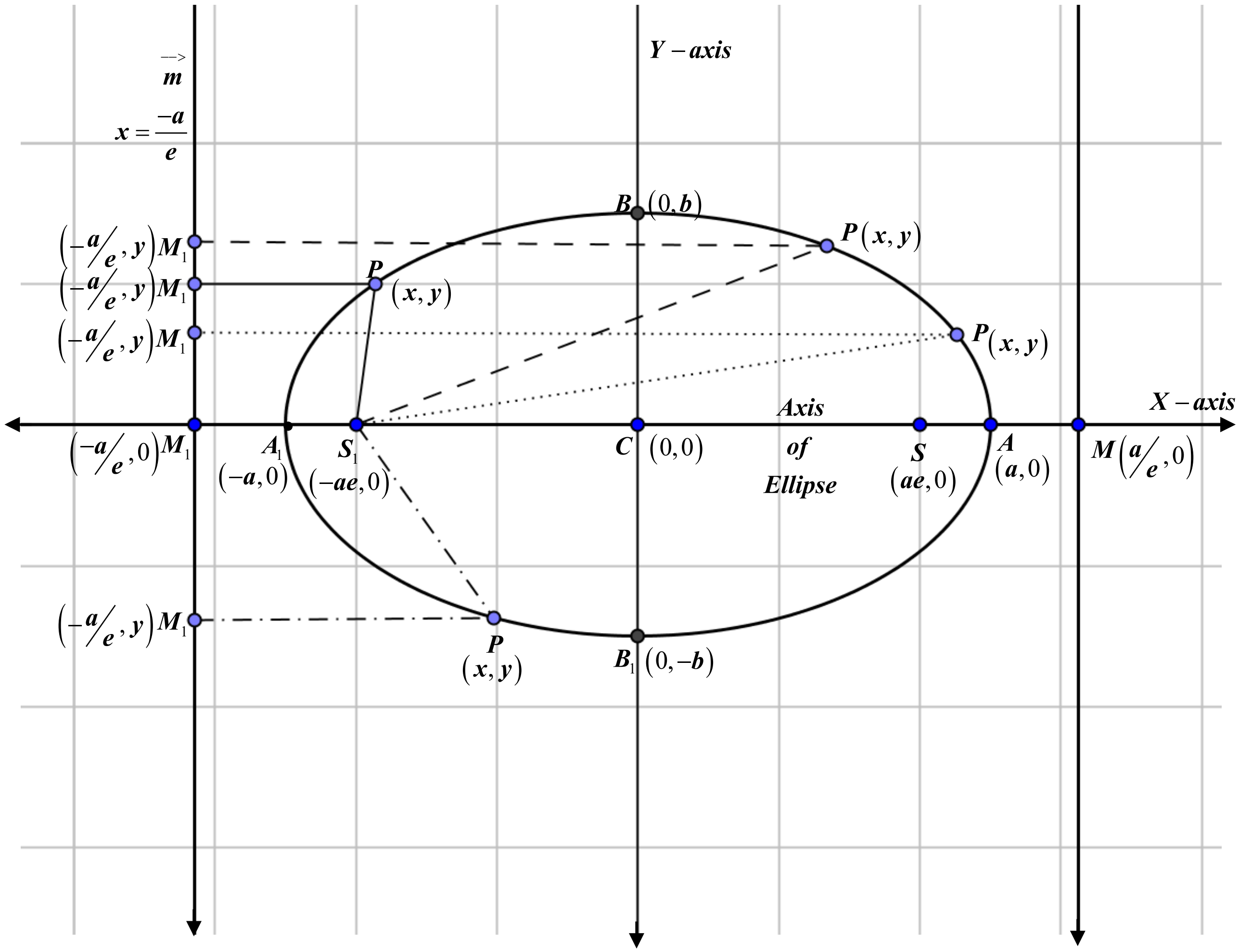
e, the *fixed ratio* is called *eccentricity*.

$$SP^2 = e^2 \times PM^2 \Rightarrow \left(x - ae\right)^2 + \left(y - 0\right)^2 = e^2\left(a/e - x\right)^2$$

$$\Rightarrow x^2 - 2aex + a^2e^2 + y^2 = e^2\left(\frac{a^2}{e^2} - 2\frac{a}{e}x + x^2\right) \Rightarrow x^2 - 2aex + a^2e^2 + y^2 = a^2 - 2aex + e^2x^2$$

$$\Rightarrow x^2\left(1 - e^2\right) + y^2 = a^2\left(1 - e^2\right) \Rightarrow \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \quad \text{where} \begin{cases} b^2 = a^2\left(1 - e^2\right) \\ b > 0 \text{ as } a > 0;\ 1 > e > 0 \end{cases}$$

↑ ↑ ↑



$S_1(-ae,0)$ is *a fixed point* called *Focus, ,* where $a > 0$.

$\overrightarrow{m} : x = \frac{-a}{e}$ is *a fixed line* called *Directrix.*

$P(x,y)$ is *a moving point* in the *same plane* as $S_1, \overrightarrow{m}$ tracing a *Ellipse.*

$M_1\left(\frac{-a}{e}, y\right)$ is the *foot of the perpendicular* from P to $\overrightarrow{m}$.

M_1 varies its position relatively with P.

P traces a *Ellipse,* such that

$$\frac{\text{distance from } P \text{ to } S_1}{\text{distance from } P \text{ to } \overrightarrow{m}} = e < 1.$$

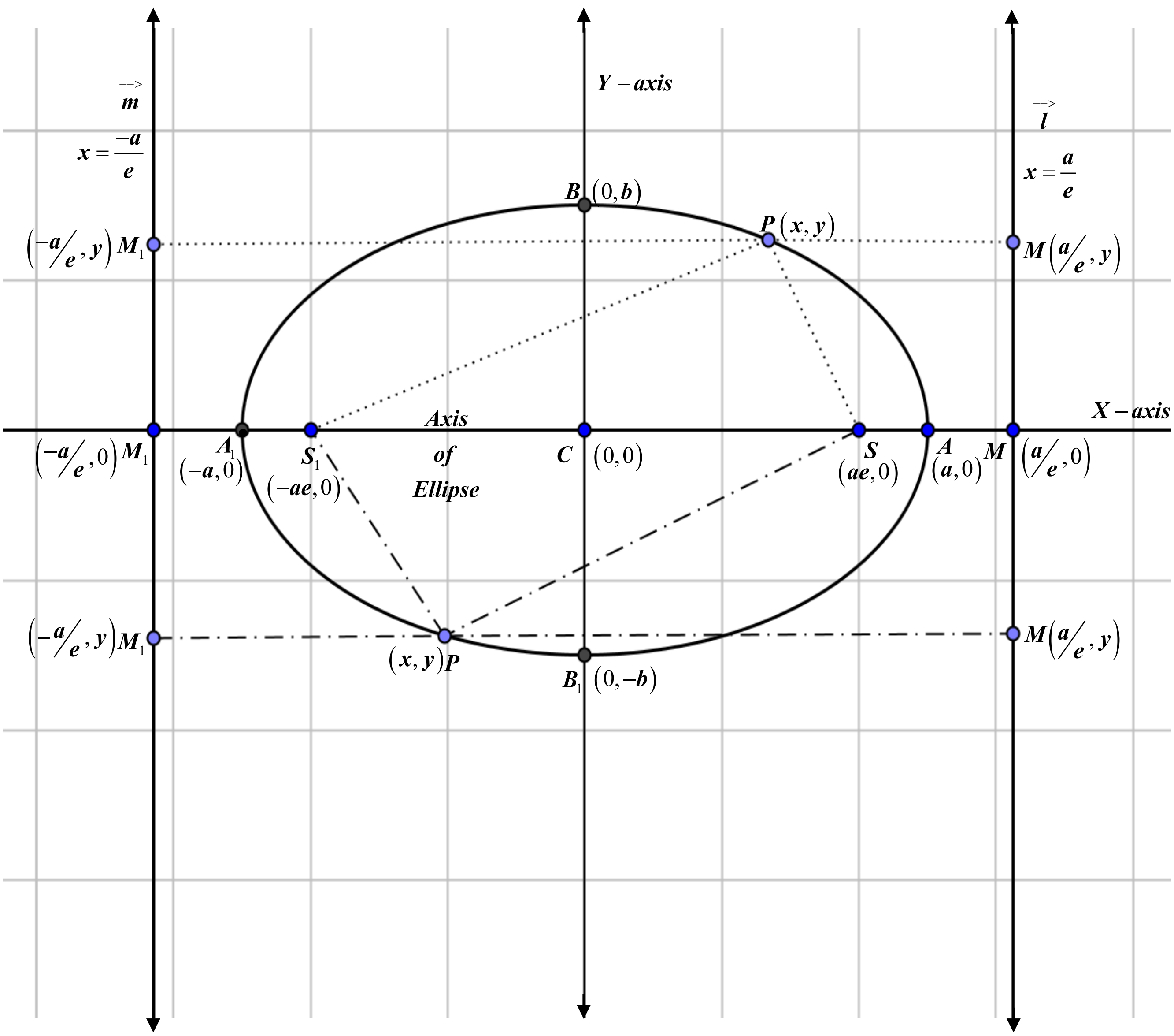
That is, $\frac{S_1P}{PM_1} = e < 1.$

e, the *fixed ratio* is called *eccentricity*.

$$S_1P^2 = e^2 \times PM_1^2 \Rightarrow \; (x+ae)^2 + (y-0)^2 = e^2\left(a/e + x\right)^2$$

$$\Rightarrow x^2 + 2aex + a^2e^2 + y^2 = e^2\left(\frac{a^2}{e^2} + 2\frac{a}{e}x + x^2\right) \Rightarrow x^2 + 2aex + a^2e^2 + y^2 = a^2 + 2aex + e^2x^2$$

$$\Rightarrow x^2\left(1-e^2\right) + y^2 = a^2\left(1-e^2\right) \Rightarrow \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \quad \text{where} \begin{cases} b^2 = a^2\left(1-e^2\right) \\ b > 0 \text{ as } a > 0;\ 1 > e > 0 \end{cases}$$



$S(ae,0), S_1(-ae,0)$ are *fixed points* called *Foci, ,* where $a > 0$.

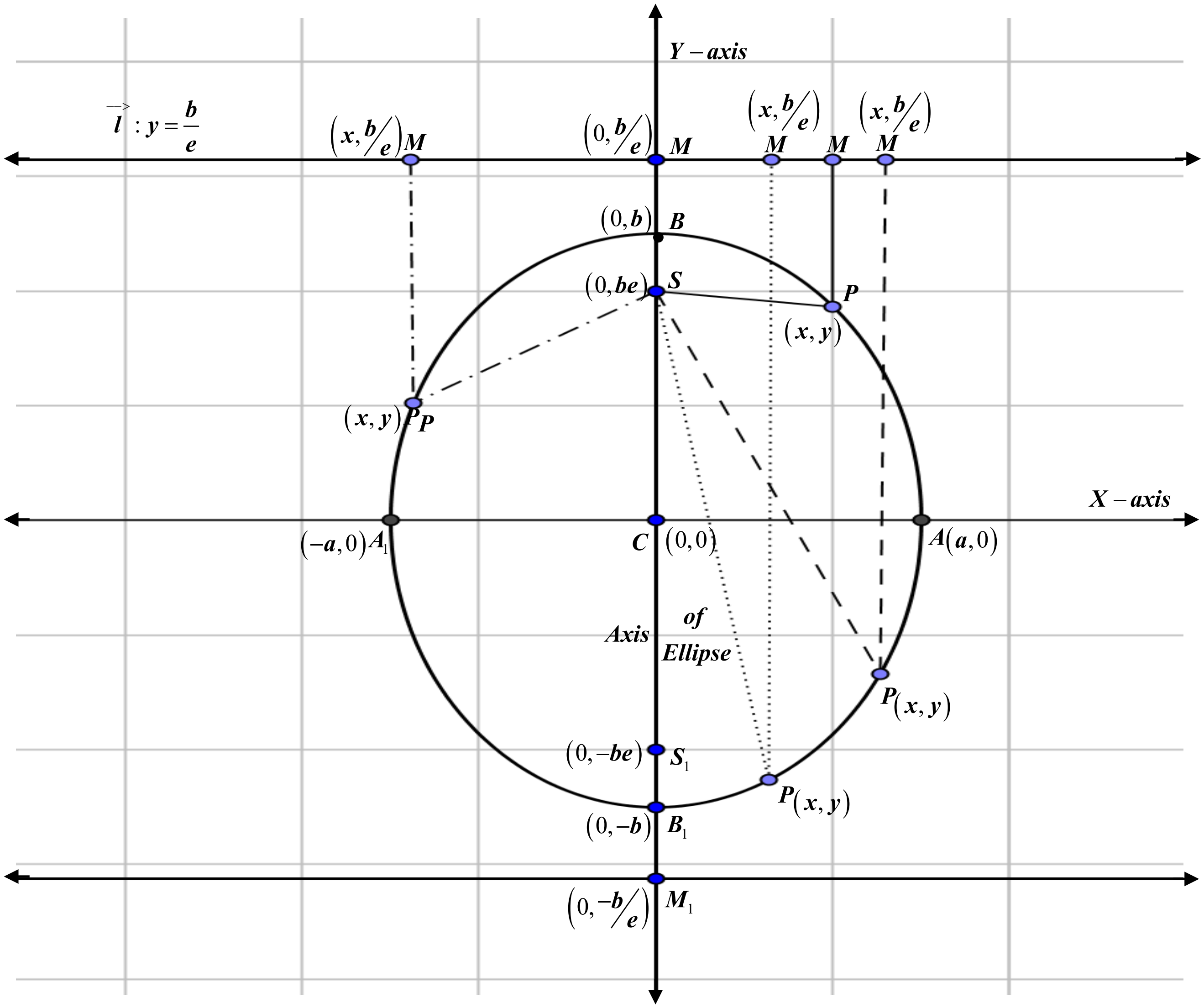
$\overrightarrow{l}: x = \frac{a}{e}, \quad \overrightarrow{m}: x = \frac{-a}{e}$ are *fixed lines* called *Directrices.*

$M\left(\frac{a}{e}, y\right), M_1\left(\frac{-a}{e}, y\right)$ are the *feet of the perpendiculars* from P to $\overrightarrow{l}, \overrightarrow{m}$ respectively.

M, M_1 varies its positions relatively with P.

$P(x,y)$ is *a moving point* traces the <u>same *Ellipse*</u>, with either pair in *the same plane* $\left(S, \overrightarrow{l}\right)$ *or* $\left(S_1, \overrightarrow{m}\right)$ such that $\frac{\text{distance from } P \text{ to } S}{\text{distance from } P \text{ to } \overrightarrow{l}} = \frac{\text{distance from } P \text{ to } S_1}{\text{distance from } P \text{ to } \overrightarrow{m}} = e < 1.$

Note that: $SP + S_1P = e.PM + e.PM_1 = e.MM_1 = e.\frac{2a}{e} = 2a$ (a constant).



$S(0,be)$ is *a fixed point* called *Focus, ,* where $b>0$.

$\vec{l}: y=\frac{b}{e}$ is *a fixed line* called *Directrix.*

$P(x,y)$ is *a moving point* in the *same plane* as $S,\ \vec{l}$ tracing a *Ellipse.*

$M\left(x,\frac{b}{e}\right)$ is the *foot of the perpendicular* from P to $\vec{l}$.

M varies its position relatively with P.

P traces a *Ellipse,* such that

$$\frac{\text{distance from } P \text{ to } S}{\text{distance from } P \text{ to } \vec{l}}=e<1.$$

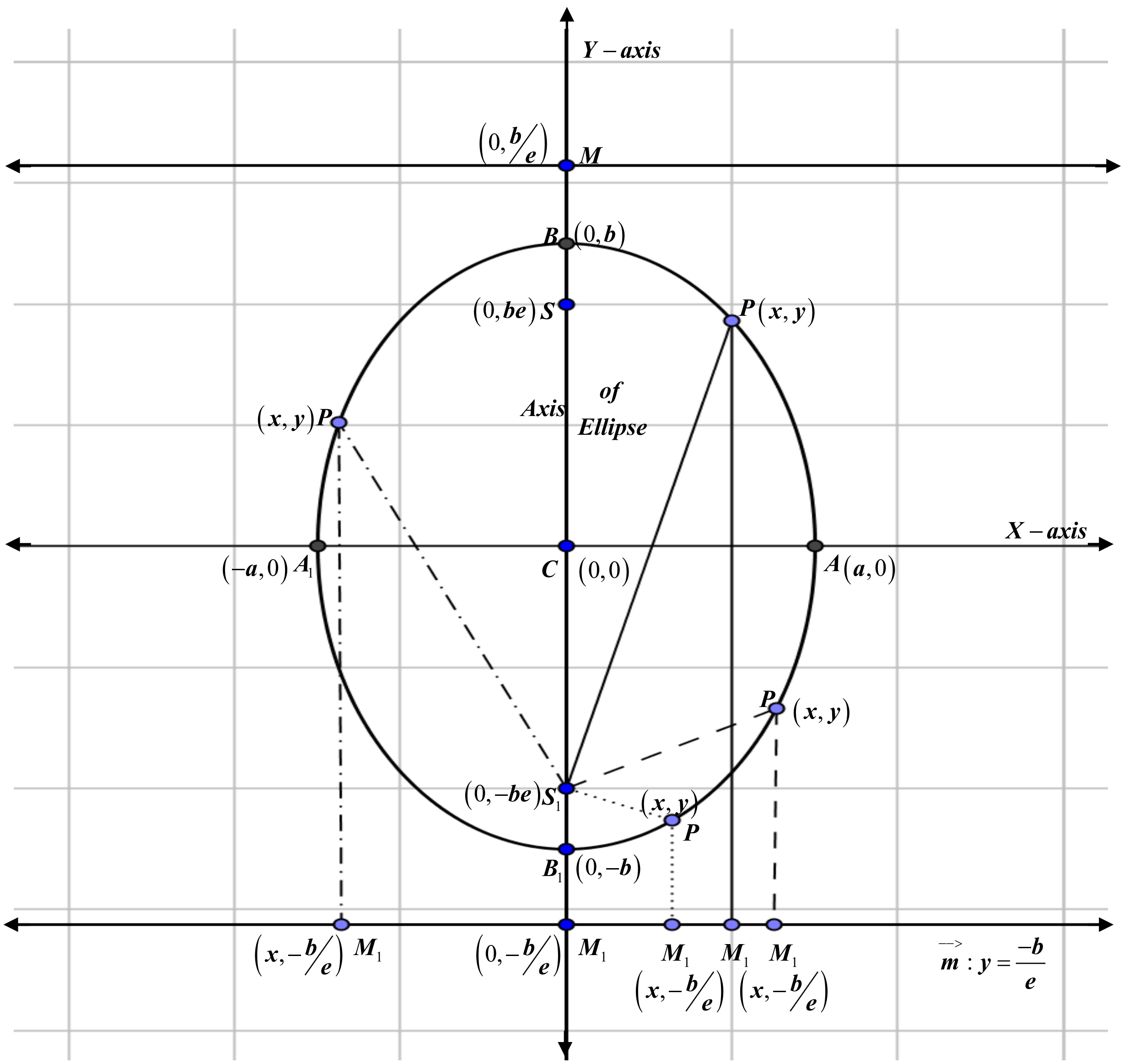
That is, $\frac{SP}{PM}=e<1$.

e, the *fixed ratio* is called *eccentricity*.

$$SP^2=e^2\times PM^2 \Rightarrow (x-0)^2+(y-be)^2=e^2\left(b/e-y\right)^2$$

$$\Rightarrow x^2+y^2-2bey+b^2e^2=e^2\left(\frac{b^2}{e^2}-2\frac{b}{e}y+y^2\right)\Rightarrow x^2+y^2-2bey+b^2e^2=b^2-2bey+e^2y^2$$

$$\Rightarrow x^2+y^2\left(1-e^2\right)=b^2\left(1-e^2\right)\Rightarrow \frac{x^2}{a^2}+\frac{y^2}{b^2}=1 \quad \text{where}\begin{cases} a^2=b^2\left(1-e^2\right) \\ a>0 \text{ as } b>0;\ 1>e>0 \end{cases}$$



$S_1(0,-be)$ is *a fixed point* called *Focus,* , where $b > 0$.

$\overrightarrow{m} : y = \frac{-b}{e}$ is *a fixed line* called *Directrix.*

$P(x,y)$ is *a moving point* in the *same plane* as $S_1, \overrightarrow{m}$ tracing a *Ellipse.*

$M_1\left(x, \frac{-b}{e}\right)$ is the *foot of the perpendicular* from P to $\overrightarrow{m}$.

M_1 varies its position relatively with P.

P traces a *Ellipse,* such that

$$\frac{\textbf{distance from } P \textbf{ to } S_1}{\textbf{distance from } P \textbf{ to } \overrightarrow{m}} = e < 1.$$

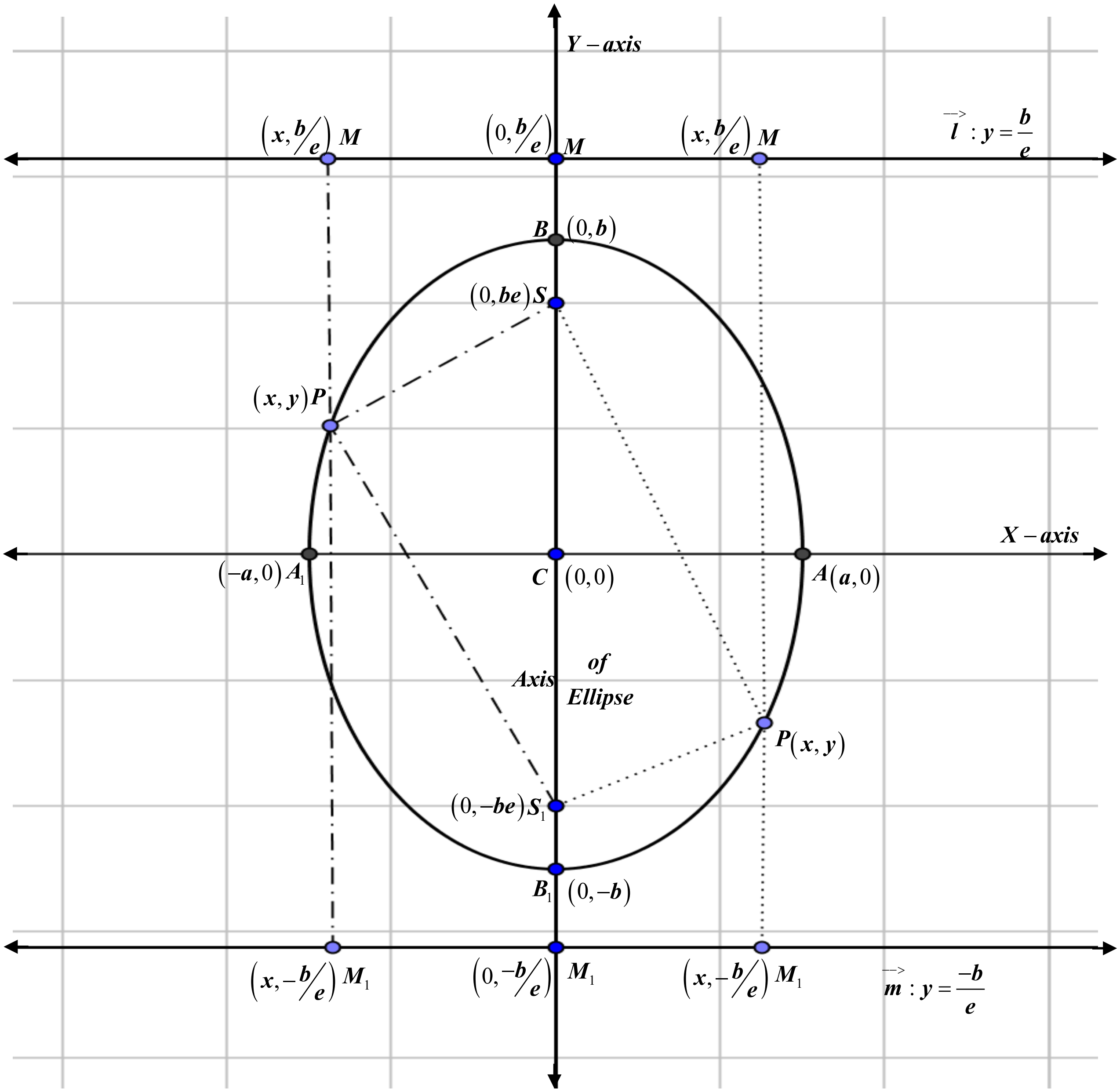
That is, $\frac{S_1P}{PM_1} = e < 1.$

e , the *fixed ratio* is called *eccentricity*.

$$S_1P^2 = e^2 \times PM_1^2 \Rightarrow (x-0)^2 + (y+be)^2 = e^2\left(b/e + y\right)^2$$

$$\Rightarrow x^2 + y^2 + 2bey + b^2e^2 = e^2\left(\frac{b^2}{e^2} + 2\frac{b}{e}y + y^2\right) \Rightarrow x^2 + y^2 + 2bey + b^2e^2 = b^2 + 2bey + e^2y^2$$

$$\Rightarrow x^2 + y^2\left(1-e^2\right) = b^2\left(1-e^2\right) \Rightarrow \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \quad \textbf{where} \begin{cases} a^2 = b^2\left(1-e^2\right) \\ a > 0 \textbf{ as } b > 0;\ 1 > e > 0 \end{cases}$$



$S(0, be), S_1(0, -be)$ are *fixed points* called *Foci, ,* where $b > 0$.

$\vec{l} : y = \frac{b}{e}, \quad \vec{m} : y = \frac{-b}{e}$ are *fixed lines* called *Directrices.*

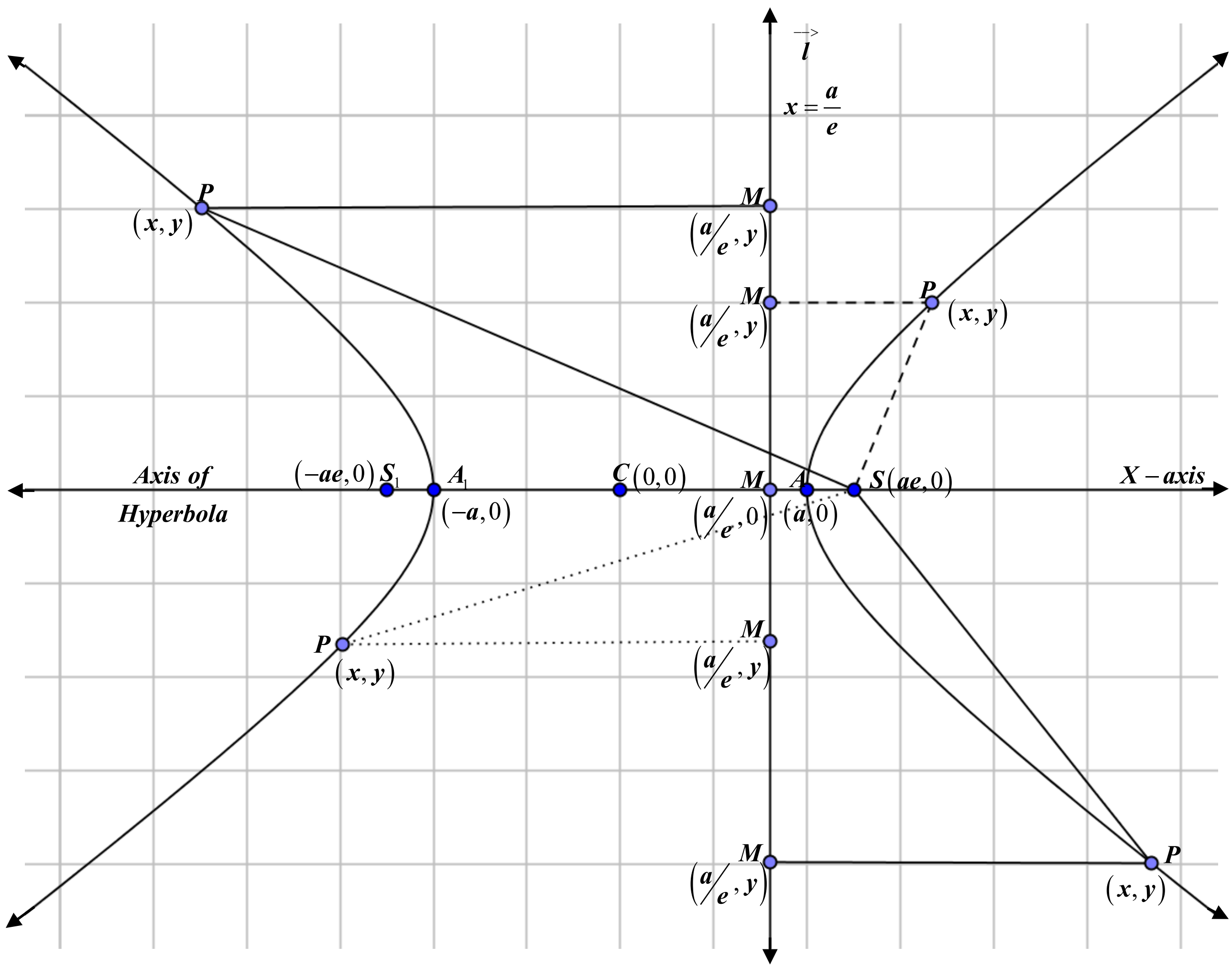
$M\left(x, \frac{b}{e}\right), M_1\left(x, \frac{-b}{e}\right)$ are the *feet of the perpendiculars* from P to $\vec{l}, \vec{m}$ respectively.

M, M_1 varies its positions relatively with P.

$P(x, y)$ is *a moving point* traces the <u>same *Ellipse*</u>, with either pair in *the same plane* $\left(S, \vec{l}\right)$ *or* $\left(S_1, \vec{m}\right)$ such that $\frac{\text{distance from } P \text{ to } S}{\text{distance from } P \text{ to } \vec{l}} = \frac{\text{distance from } P \text{ to } S_1}{\text{distance from } P \text{ to } \vec{m}} = e < 1.$

Note that: $SP + S_1P = e.PM + e.PM_1 = e.MM_1 = e.\frac{2b}{e} = 2b$ (a constant).

Basic Forms of Hyperbolas



$S(ae, 0)$ is *a fixed point* called *Focus,* , where $a > 0$.

$\vec{l} : x = \frac{a}{e}$ is *a fixed line* called *Directrix.*

$P(x, y)$ is *a moving point* in the *same plane* as $S, \vec{l}$ tracing a *Hyperbola.*

$M\left(\frac{a}{e}, y\right)$ is the *foot of the perpendicular* from P to $\vec{l}$.

P traces a *Hyperbola,* such that

$$\frac{\text{distance from } P \text{ to } S}{\text{distance from } P \text{ to } \vec{l}} = e > 1.$$

That is, $\frac{SP}{PM} = e > 1$.

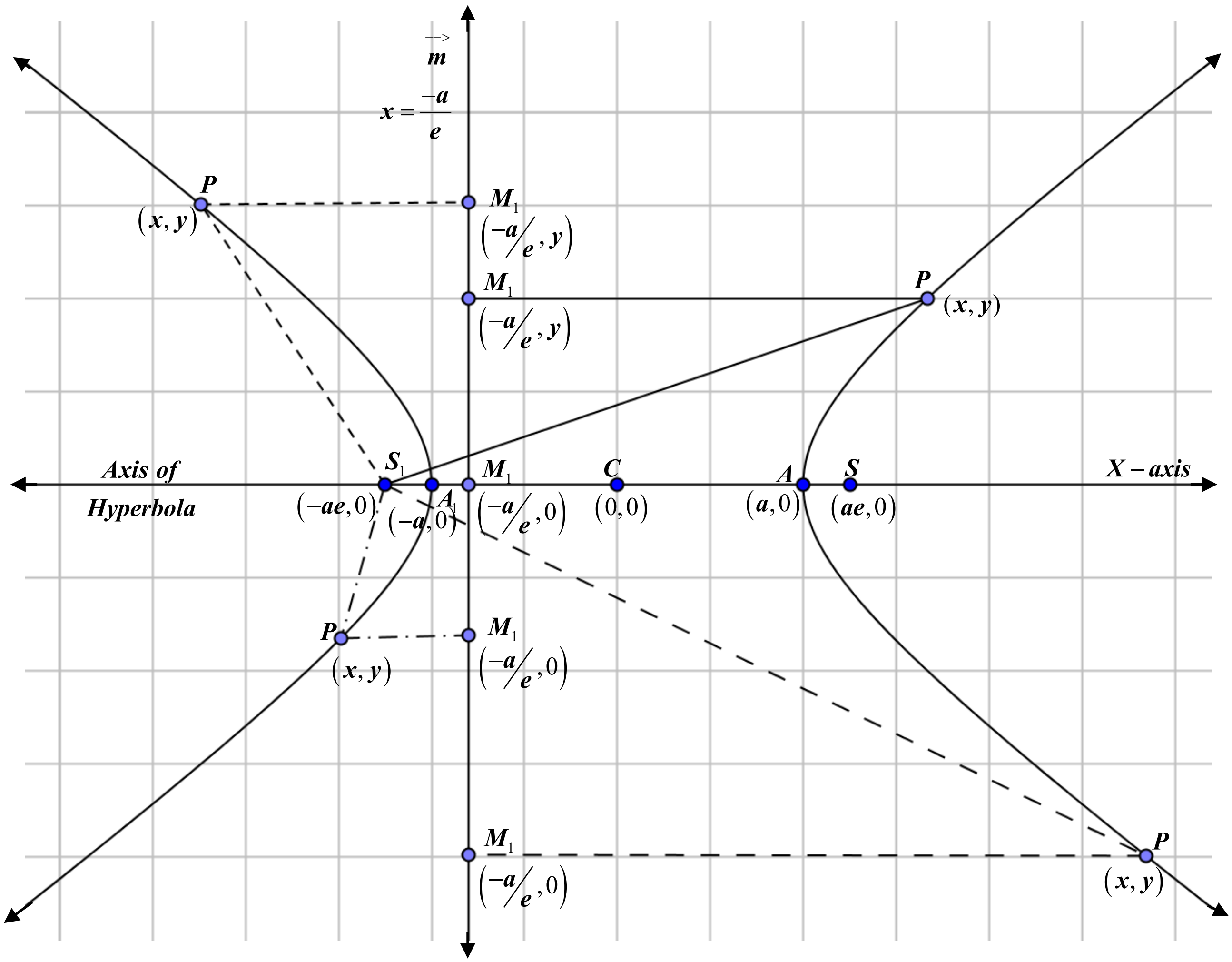
e, the *fixed ratio* is called *eccentricity.*

M varies its position relatively with P .

$$SP^2 = e^2 \times PM^2 \Rightarrow (x - ae)^2 + (y - 0)^2 = e^2 \left(a/e - x\right)^2$$

$$\Rightarrow x^2 - 2aex + a^2e^2 + y^2 = e^2\left(\frac{a^2}{e^2} - 2\frac{a}{e}x + x^2\right) \Rightarrow x^2 - 2aex + a^2e^2 + y^2 = a^2 - 2aex + e^2x^2$$

$$\Rightarrow x^2(1-e^2) + y^2 = a^2(1-e^2) \Rightarrow \frac{x^2}{a^2} - \frac{y^2}{b^2} = 1 \quad \text{where} \begin{cases} b^2 = a^2(e^2-1) \\ b > 0 \text{ as } a > 0;\ e > 1 \end{cases}$$



$S_1(-ae, 0)$ is *a fixed point* called *Focus, ,* where $a > 0$.

$\overrightarrow{m} : x = \frac{-a}{e}$ is *a fixed line* called *Directrix.*

$P(x, y)$ is *a moving point* in the *same plane* as $S_1, \overrightarrow{m}$ tracing a *Hyperbola.*

$M_1\left(\frac{-a}{e}, y\right)$ is the *foot of the perpendicular* from P to $\overrightarrow{m}$.

M_1 varies its position relatively with P.

P traces a *Hyperbola,* such that

$$\frac{\textbf{distance from } P \textbf{ to } S_1}{\textbf{distance from } P \textbf{ to } \overrightarrow{m}} = e > 1.$$

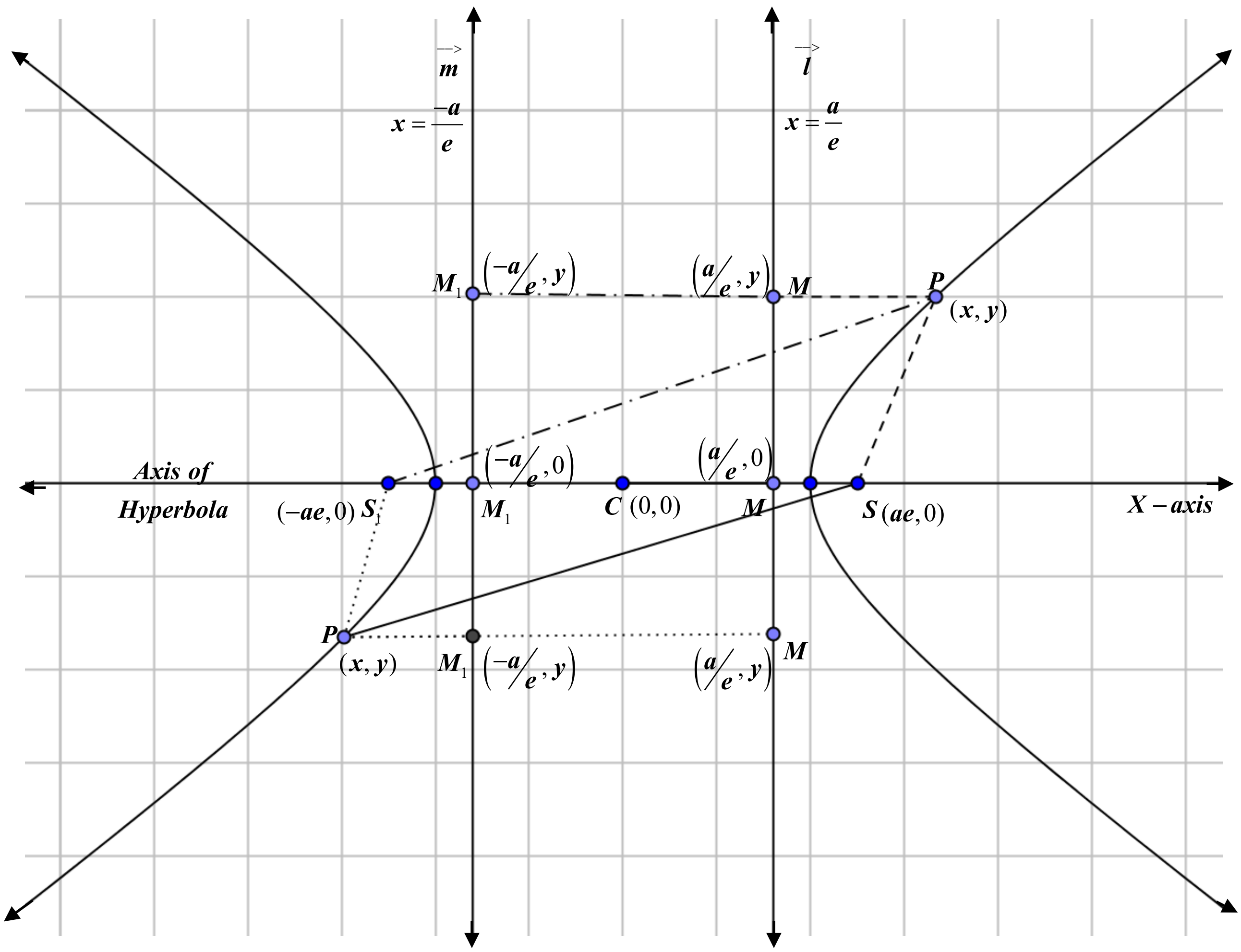
That is, $\frac{S_1P}{PM_1} = e > 1.$

e, the *fixed ratio* is called *eccentricity*.

$$S_1P^2 = e^2 \times PM_1^{\ 2} \Rightarrow \ (x + ae)^2 + (y - 0)^2 = e^2\left(a/e + x\right)^2$$

$$\Rightarrow x^2 + 2aex + a^2e^2 + y^2 = e^2\left(\frac{a^2}{e^2} + 2\frac{a}{e}x + x^2\right) \Rightarrow x^2 + 2aex + a^2e^2 + y^2 = a^2 + 2aex + e^2x^2$$

$$\Rightarrow x^2\left(1 - e^2\right) + y^2 = a^2\left(1 - e^2\right) \Rightarrow \frac{x^2}{a^2} - \frac{y^2}{b^2} = 1 \quad \textbf{where} \begin{cases} b^2 = a^2\left(e^2 - 1\right) \\ b > 0 \ \textbf{ as } \ a > 0; \ e > 1 \end{cases}$$



$S(ae,0), S_1(-ae,0)$ are *fixed points* called *Foci,* , where $a>0$.

$\overrightarrow{l}: x=\frac{a}{e}$, $\overrightarrow{m}: x=\frac{-a}{e}$ are *fixed lines* called *Directrices.*

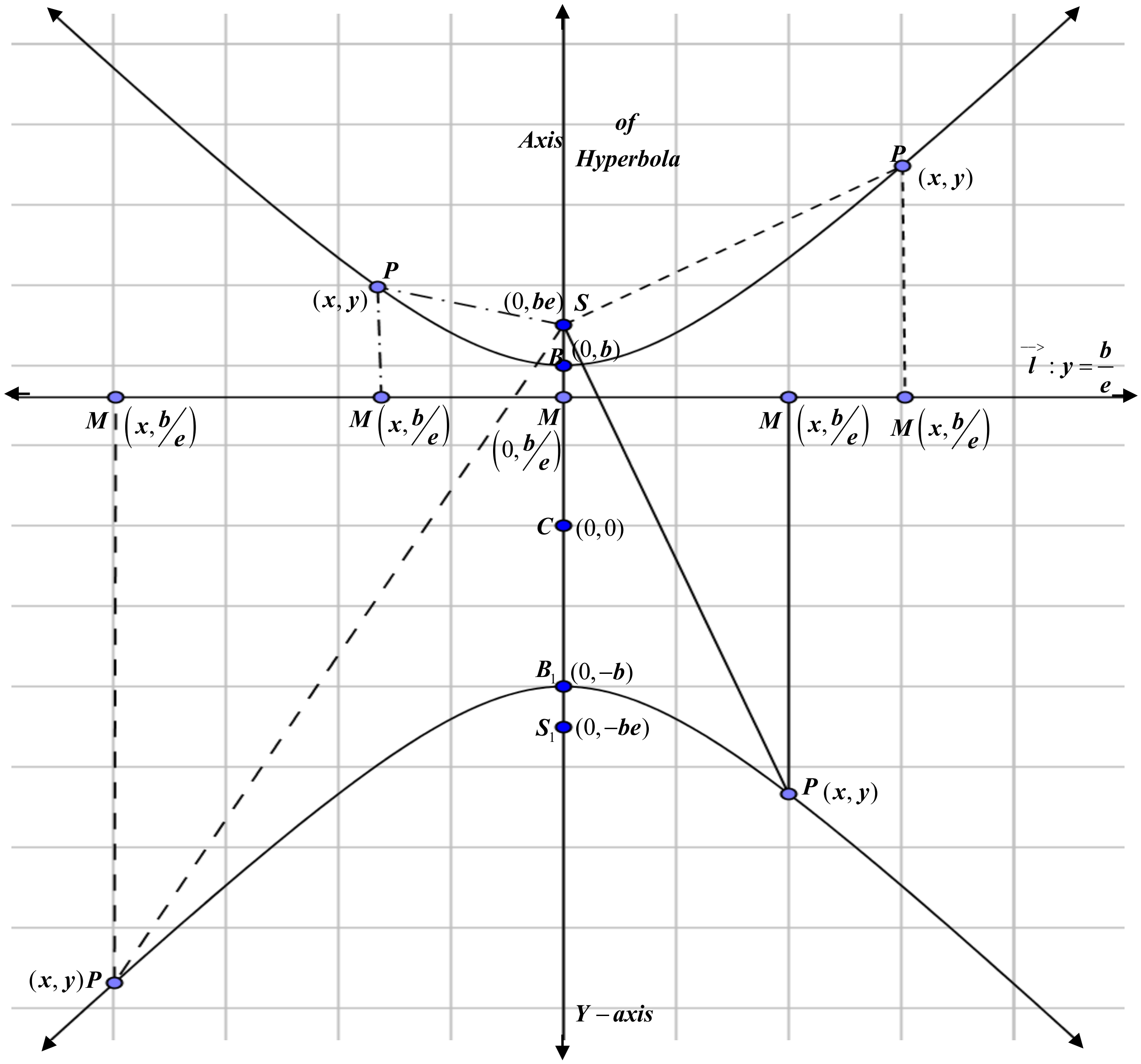
$M\left(\frac{a}{e}, y\right), M_1\left(\frac{-a}{e}, y\right)$ are the *feet of the perpendiculars* from P to $\overrightarrow{l}, \overrightarrow{m}$ respectively.

M, M_1 varies its positions relatively with P.

$P(x,y)$ is *a moving point* traces the <u>same *Hyperbola*,</u> with either pair in *the same plane* $\left(S, \overrightarrow{l}\right)$ *or* $\left(S_1, \overrightarrow{m}\right)$ such that $\frac{\text{distance from } P \text{ to } S}{\text{distance from } P \text{ to } \overrightarrow{l}} = \frac{\text{distance from } P \text{ to } S_1}{\text{distance from } P \text{ to } \overrightarrow{m}} = e > 1.$

Note that: $SP - S_1P = e.PM - e.PM_1 = \pm e.MM_1 = \pm e.\frac{2a}{e} = \pm 2a$ (a constant).

$\therefore \left|SP - S_1P\right| = 2a$ (a constant).



$S(0, be)$ is *a fixed point* called *Focus,* , where $b > 0$.

$\vec{l} : y = \frac{b}{e}$ is *a fixed line* called *Directrix.*

$P(x, y)$ is *a moving point* in the *same plane* as $S, \vec{l}$ tracing a *Hyperbola.*

$M\left(x, \frac{b}{e}\right)$ is the *foot of the perpendicular* from P to $\vec{l}$.

M varies its position relatively with P.

P traces a *Hyperbola,* such that

$$\frac{\textbf{distance from } P \textbf{ to } S}{\textbf{distance from } P \textbf{ to } \vec{l}} = e > 1.$$

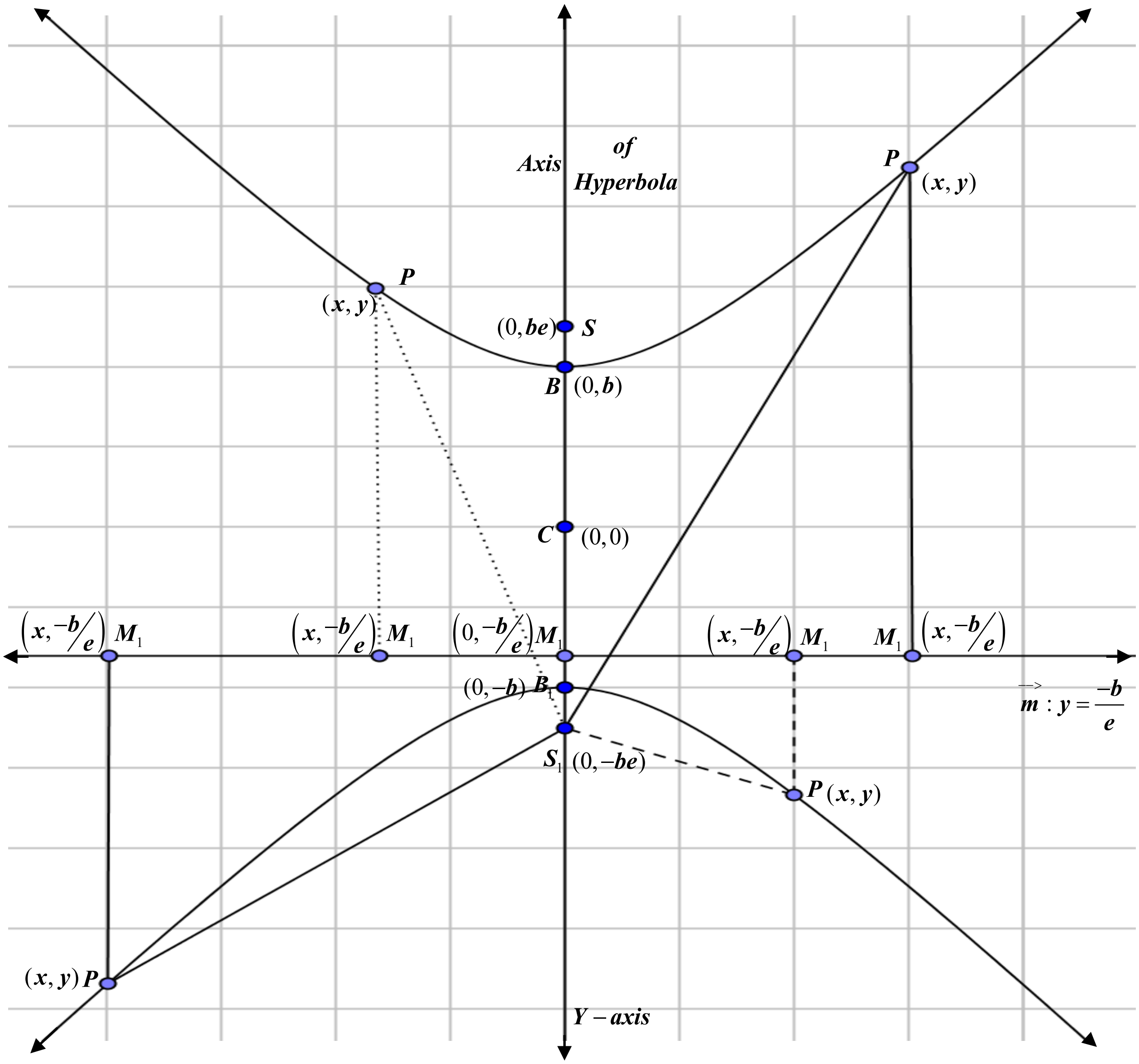
That is, $\frac{SP}{PM} = e > 1$.

e, the *fixed ratio* is called *eccentricity.*

$$SP^2 = e^2 \times PM^2 \Rightarrow (x-0)^2 + (y-be)^2 = e^2\left(b/e - y\right)^2$$

$$\Rightarrow x^2 + y^2 - 2bey + b^2e^2 = e^2\left(\frac{b^2}{e^2} - 2\frac{b}{e}y + y^2\right) \Rightarrow x^2 - 2bey + b^2e^2 + y^2 = b^2 - 2bey + e^2y^2$$

$$\Rightarrow y^2\left(1-e^2\right) + x^2 = b^2\left(1-e^2\right) \Rightarrow \frac{y^2}{b^2} - \frac{x^2}{a^2} = 1 \quad \text{where} \begin{cases} a^2 = b^2\left(e^2-1\right) \\ a > 0 \text{ as } b > 0;\ e > 1 \end{cases}$$



$S_1(0,-be)$ is *a fixed point* called *Focus,* , where $b>0$.

$\vec{m}: y=\dfrac{-b}{e}$ is *a fixed line* called *Directrix.*

$P(x,y)$ is *a moving point* in the *same plane* as $S_1, \vec{m}$ tracing a *Hyperbola.*

$M_1\left(x,\dfrac{-b}{e}\right)$ is the *foot of the perpendicular* from P to $\vec{m}$.

M_1 varies its position relatively with P.

P traces a *Hyperbola,* such that

$$\frac{\textbf{distance from } P \textbf{ to } S_1}{\textbf{distance from } P \textbf{ to } \vec{m}}=e>1.$$

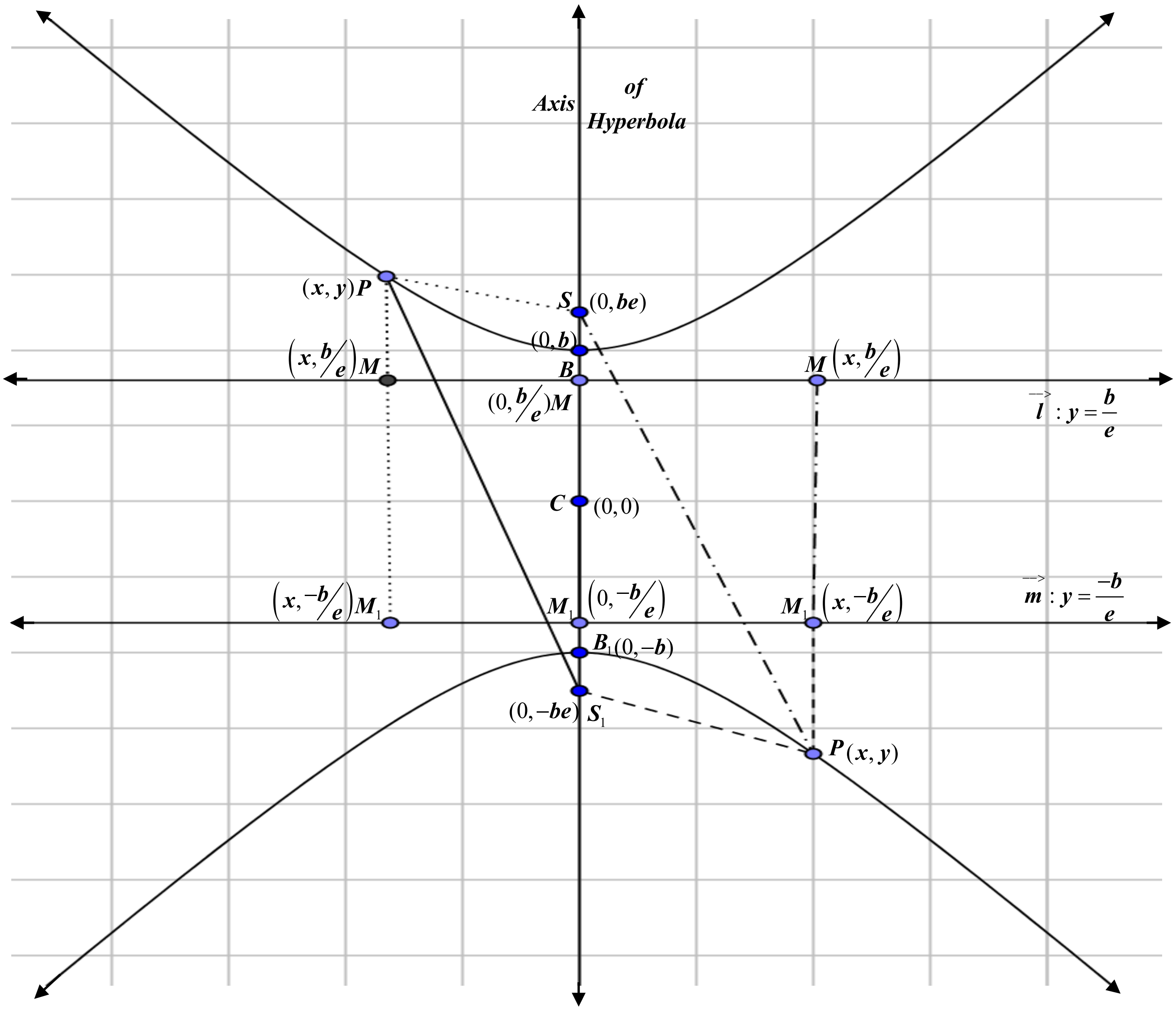
That is, $\dfrac{S_1P}{PM_1}=e>1$.

e, the *fixed ratio* is called *eccentricity*.

$$S_1P^2=e^2\times PM_1^{\,2}\Rightarrow (x-0)^2+(y+be)^2=e^2\left(b/e+y\right)^2$$

$$\Rightarrow x^2+y^2+2bey+b^2e^2=e^2\left(\frac{b^2}{e^2}+2\frac{b}{e}y+y^2\right)\Rightarrow x^2+2bey+b^2e^2+y^2=b^2+2bey+e^2y^2$$

$$\Rightarrow y^2\left(1-e^2\right)+x^2=b^2\left(1-e^2\right)\Rightarrow \frac{y^2}{b^2}-\frac{x^2}{a^2}=1 \quad \textbf{where} \begin{cases} a^2=b^2\left(e^2-1\right) \\ a>0 \ \textbf{ as } \ b>0;\ e>1 \end{cases}$$



$S(0,be), S_1(0,-be)$ are *fixed points* called *Foci,* , where $b>0$.

$\vec{l}: y=\frac{b}{e}$, $\vec{m}: y=\frac{-b}{e}$ are *fixed lines* called *Directrices.*

$M\left(x,\frac{b}{e}\right), M_1\left(x,\frac{-b}{e}\right)$ are the *feet of the perpendiculars* from P to $\vec{l}, \vec{m}$ respectively.

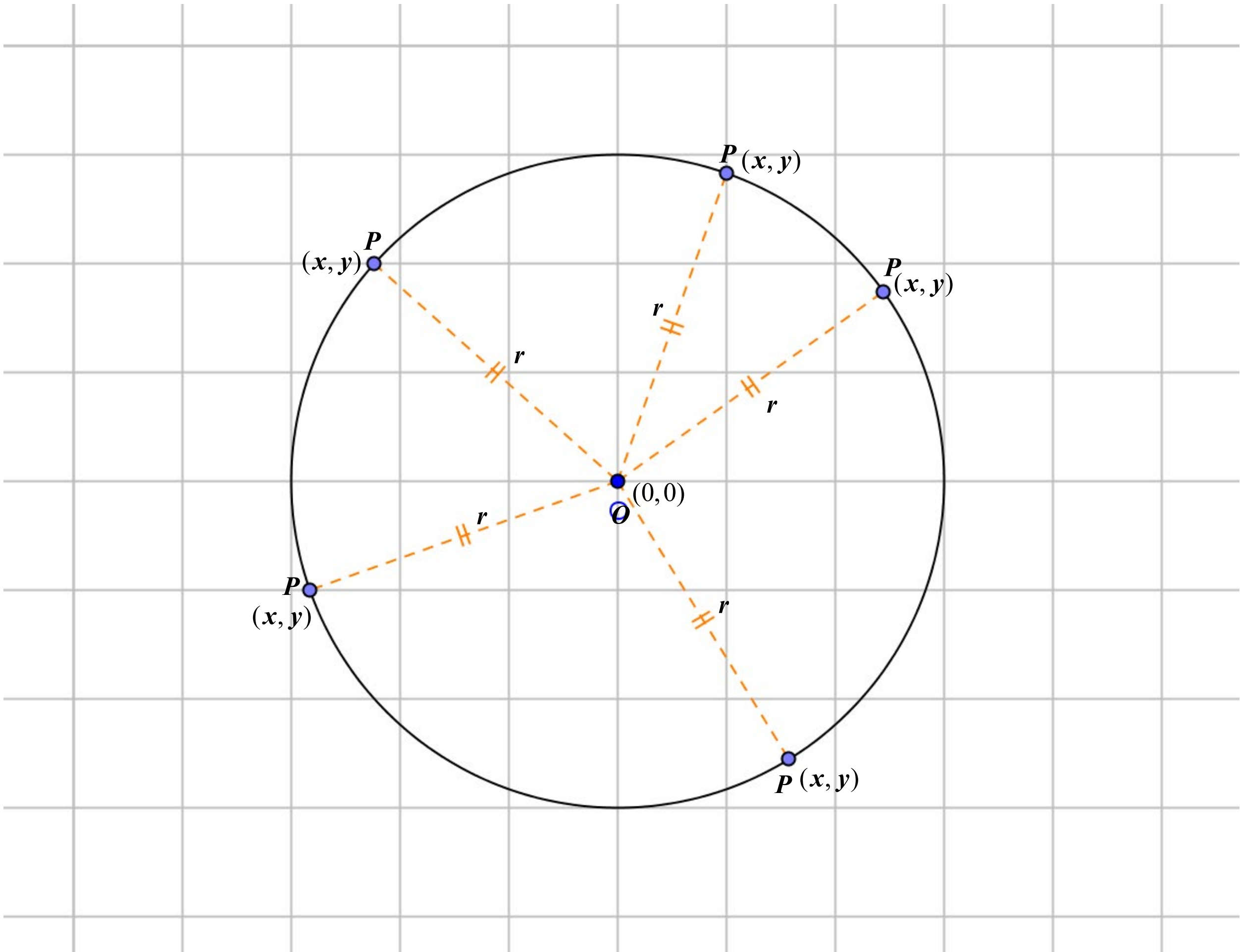
M, M_1 varies its positions relatively with P.

$P(x,y)$ is *a moving point* traces the <u>same *Hyperbola*</u>, with either pair in *the same plane* $\left(S,\vec{l}\right)$ *or* $\left(S_1,\vec{m}\right)$ such that $\frac{\text{distance from } P \text{ to } S}{\text{distance from } P \text{ to } \vec{l}} = \frac{\text{distance from } P \text{ to } S_1}{\text{distance from } P \text{ to } \vec{m}} = e > 1.$

Note that: $SP - S_1P = e.PM - e.PM_1 = \pm e.MM_1 = \pm e.\frac{2b}{e} = \pm 2b$ (a constant).

$\therefore |SP - S_1P| = 2b$ (a constant).

Basic Forms of Circles



$O(0,0)$ is *a fixed point* called *Centre*.

$P(x,y)$ is *a moving point* in the same plane as O, tracing a *Circle.*

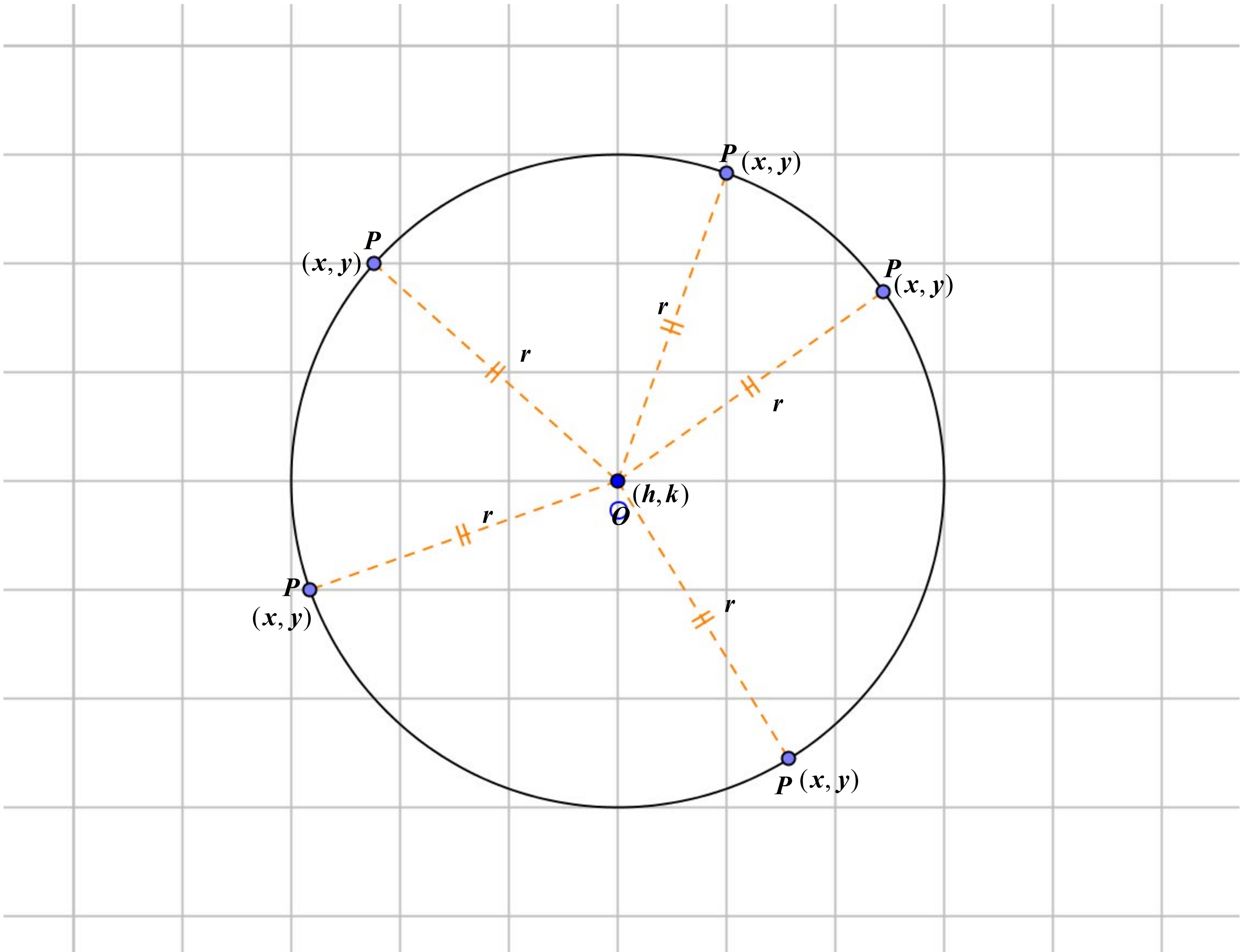
P traces a *Circle,* such that

distance from P to O $= r$ (a constant).

r is called the *Radius* of the *Circle*.

$$OP^2 = r^2 \Rightarrow (x-0)^2 + (y-0)^2 = r^2$$

$$\Rightarrow x^2 + y^2 = r^2$$



$O(h,k)$ is *a fixed point* called *Centre*.

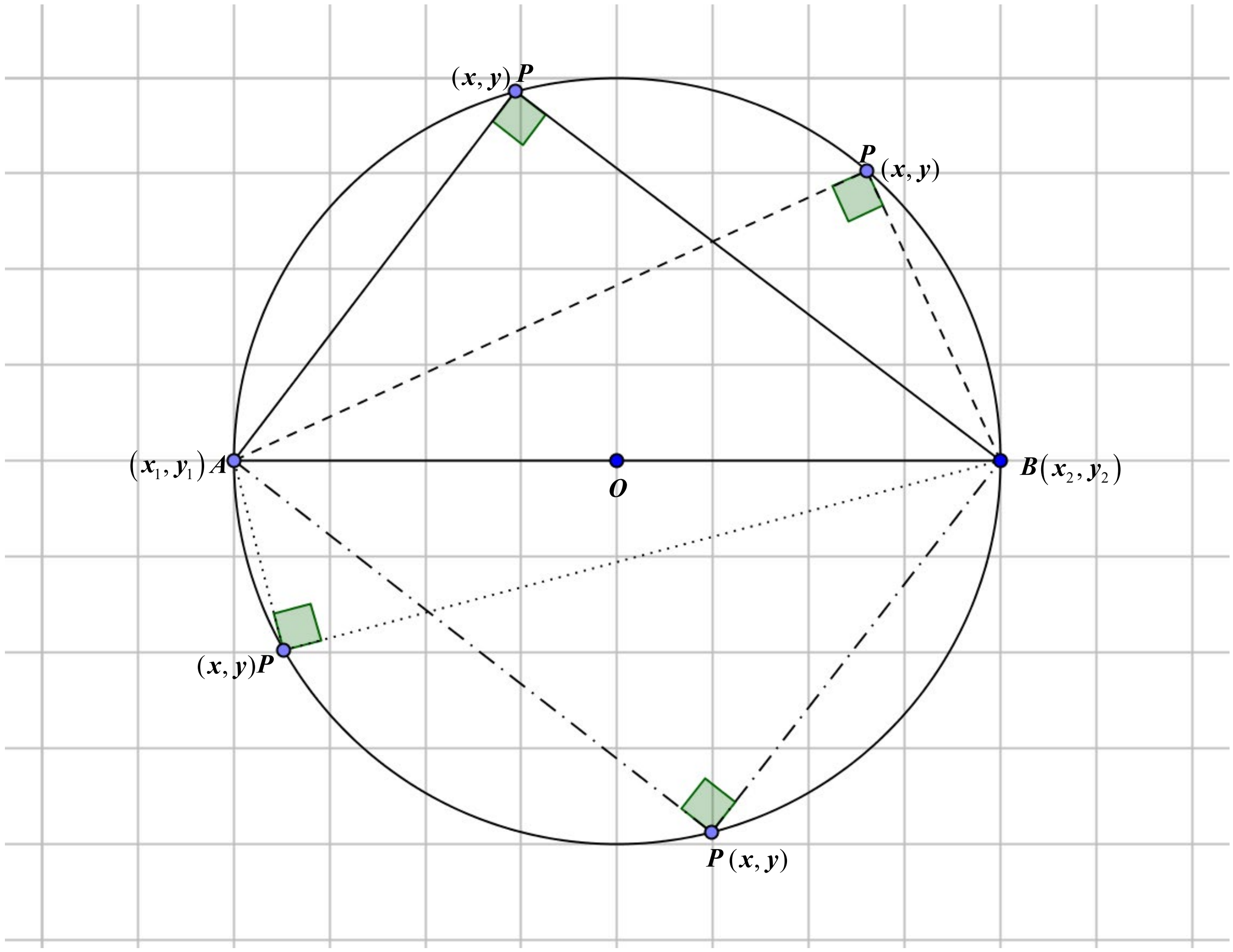
$P(x,y)$ is *a moving point* in the same plane as O, tracing a *Circle.*

P traces a *Circle,* such that

distance from P to O $= r$ (a constant).

r is called the *Radius* of the *Circle*.

$$OP^2 = r^2 \Rightarrow (x-h)^2 + (y-k)^2 = r^2.$$



$\boldsymbol{O}$ is *a fixed point* called *Centre*.

$\boldsymbol{A}(\boldsymbol{x}_1, \boldsymbol{y}_1), \boldsymbol{B}(\boldsymbol{x}_2, \boldsymbol{y}_2)$ are *diameter ends of a Circle.*

$\boldsymbol{P}(\boldsymbol{x}, \boldsymbol{y})$ is *a moving point* in the same plane as $\boldsymbol{O}, \boldsymbol{A}, \boldsymbol{B}$ tracing a *Circle.*

$\boldsymbol{P}$ traces a *Circle,* such that $\measuredangle APB = 90^\circ$ **.**

$\measuredangle APB = 90^\circ \Rightarrow PA \perp PB \Rightarrow$ **slope of AP** $\times$ **slope of BP =** -1

$$\Rightarrow \frac{y - y_1}{x - x_1} \cdot \frac{y - y_2}{x - x_2} = -1 \quad \Rightarrow \quad (y - y_1).(y - y_2) = -(x - x_1).(x - x_2)$$

$$\Rightarrow \quad (x - x_1).(x - x_2) + (y - y_1).(y - y_2) = 0$$

All The Best